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Institute of Numerical Methods and Mechanical Engineering
Technische Universität Darmstadt
64289 Darmstadt
Tel: +49 6151 16 -75840
Fax: +49 6151 16 – 4479
iutam2015@dyn.tu-darmstadt.de
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Numerical Continuation Applied to Nonlinear Rotor Dynamics
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Monday Morning, July 6, 2015
The classical analysis of dissipative dynamics focuses on finding equilibria and periodic solutions and their stability. Considering systems with two or more degrees of freedom, such as interacting nonlinear oscillators, this analysis must be supplemented by a study of quasi-periodic solutions and their bifurcations. Geometrically, quasi-periodic solutions correspond with tori, for instance two- or three-frequency oscillations describe motion on a torus. What do we gain from geometric insight? It turns out that geometric pictures improve our understanding of the basic dynamics and in particular, bifurcational changes of the dynamics correspond with changes in the geometry and vice versa. The purpose of this lecture is to demonstrate this for a few relatively simple models of interacting oscillators. The surprise is that these models already contain a striking amount of complexity. The results agree with a visionary paper by Ruelle and Takens (1971), where the relation is suggested between turbulence in fluids and the bifurcation scenario in which equilibrium produces periodic solution, subsequently by bifurcation leading to a torus which in the next bifurcation produces a strange attractor.

Consider first a system of two nonlinearly coupled oscillators:

\[
\begin{align*}
\ddot{x} + \delta x^2 \dot{x} + x + \gamma x^3 + axy &= 0, \\
\ddot{y} + \kappa \dot{y} + 4y + bx^2 &= 0.
\end{align*}
\]  
(1)

Using averaging, continuation and numerical bifurcation techniques in this case of the internal resonance 1 : 2, we find a 2π-periodic solution which undergoes a supercritical Neimark-Sacker bifurcation, yielding a stable torus. Choosing a route in parameter space, we show by numerical bifurcation techniques how the torus gets destroyed by dynamical and topological changes in the involved manifolds. The 1 : 6-resonance turns out to be prominent in parameter space and we detected a cascade of period doublings within the corresponding resonance tongue yielding a strange attractor. The phenomena agree with the Ruelle-Takens scenario.

Other periodic regimes are present in this system and there is interesting evidence that two different regimes interact with each other, yielding yet another type of strange attractor. In this context certain π-periodic solutions emerge that are studied by continuation following the Poincaré-Lindstedt method using Mathieu-functions; when the implicit
function theorem breaks down, the analysis is supplemented by numerical bifurcation tech-
niques.

As a second problem we consider a self-excited as well as parametrically excited three-
mass chain system (a Tondl model) in 1 : 2 : 3 resonance. The engineering context is as
follows. Consider a three-mass system where the middle mass \( m_2 \) is self-excited by flow,
which is modeled by a Rayleigh term. The masses \( m_1 \) and \( m_3 \) are coupled to \( m_2 \) and
can be parametrically excited. If one wishes to quench the motion of the self-excited mass
\( m_2 \), with a suitable tuning, the masses \( m_1 \) and \( m_2 \) can be used as energy absorbers. The
equations of motion are as follows:

\[
\begin{align*}
    m_1 \ddot{y}_1 + b \dot{y}_1 + k_0 (1 + \varepsilon \cos \omega t) y_1 - k_1 (y_2 - y_1) &= 0, \\
    m_2 \ddot{y}_2 - \beta_0 U^2 (1 - \gamma_0 y_2^2) \dot{y}_2 + 2k_1 y_2 - k_1 (y_1 + y_3) &= 0, \\
    m_3 \ddot{y}_3 + b \dot{y}_3 + k_0 (1 + \varepsilon \cos \omega t) y_3 - k_1 (y_2 - y_3) &= 0.
\end{align*}
\]

(2)

Here \( y_1, y_2 \) and \( y_3 \) represent the deflections of the masses \( m_1, m_2 \) and \( m_3 \) respectively.
The parameters \( k_0, k_1, \gamma_0 \) are assumed to be positive. The parameter \( \varepsilon \) is assumed to be
small and positive. The Rayleigh excitation parameter \( \beta_0 \) and the damping coefficient \( b \)
are assumed to be \( O(\varepsilon) \), \( b = \varepsilon \kappa \) and \( \beta_0 = \varepsilon \beta \), whereas all the remaining parameters are
constants which do not depend on \( \varepsilon \).

For the analysis both averaging-normalization and numerical simulations are used. A
set of necessary and sufficient conditions is given for the general system to be in 1 : 2 : 3
resonance. Using averaging-normalization, we are able to locate various periodic solutions.
A bifurcation diagram is produced for each of the resonances generated by the quasi-
periodic solutions, revealing interesting dynamics like a stable 2-torus, torus doubling and
in the neighborhood of a Hopf-Hopf bifurcation a stable 3-torus. Typical bifurcations are
Neimark-Sacker, Saddle-Node, Chenciner bifurcation (degenerate Hopf), Fold-Neimark-
Sacker bifurcation, Branching point bifurcation.

The tori eventually break up, leading to strange attractors and chaos. Comparing the
results of averaging-normalization with the dynamics of the original system shows good
agreement. The bifurcation diagram of the normal form shows a complex accumulation of
period doublings.
Abstract

Evolutionary dynamics combines game theory and nonlinear dynamics to model competition in biological and social situations. The replicator equation is a standard paradigm in evolutionary dynamics [1],

$$\dot{x}_i = x_i (f_i(x) - \phi), \quad i = 1, \ldots, n$$

(1)

where $x_i$ is the frequency of strategy $i$, $f_i$ is its fitness, and $\phi = \sum x_i f_i$ is the average fitness. In words, equation (1) says that the growth rate of each strategy’s frequency is its excess fitness, i.e. the deviation of its fitness from the average. The game-theoretic aspect of the model lies in the choice of fitness function, which is determined by a payoff matrix $A$:

$$f_i = (Ax)_i.$$  

(2)

Previous work by Ruelas and Rand [2] investigated the Rock-Paper-Scissors replicator dynamics problem with periodic forcing of the payoff coefficients. This work extends the previous to consider the case of quasiperiodic forcing. That is, the payoff matrix is

$$A = \begin{pmatrix} 0 & -1 - F(t) & 1 + F(t) \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

(3)

where the forcing function $F$ is given by

$$F(t) = \epsilon \left((1 - \delta) \cos \omega_1 t + \delta \cos \omega_2 t \right).$$

(4)

This model may find applications in biological or social systems where competition is affected by cyclical processes on different scales, such as days/years or weeks/years.

The methods used in this paper to study the quasiperiodically forced Rock-Paper-Scissors system (1)-(4) include (i) numerical simulation, (ii) computation of Lyapunov exponents, and (iii) harmonic balance to approximate the periodic solutions predicted by Floquet theory. We investigate the linear stability of the interior equilibrium point, where all three strategies coexist. We find that stability depends sensitively on the frequencies $\omega_1$ and $\omega_2$, and that the region of instability in the $\omega_1 - \omega_2$ plane exhibits self-similarity. As predicted by perturbation analysis [3], there are tongues of instability along the lines

$$n \omega_1 + m \omega_2 = 2/\sqrt{3}$$

(5)

for all pairs of integers $(n, m)$.

It is shown that the results of the three methods are mutually consistent.
Figure 1: (Top) Transition curves in the $\omega_1 - \omega_2$ plane predicted by harmonic balance. (Bottom left) Region of instability given by numerical integration. (Bottom right) Contour plot of Lyapunov exponents. For all plots, $\varepsilon = 1.3, \delta = 0.6$.

References


An Analytical Dynamics Approach to the Energy Control of General Nonlinear Lattices

Firdaus E. Udwadia*, Harshavardhan Mylapilli*

* Departments of Aerospace and Mechanical Engineering, Civil Engineering, Mathematics, and Information and Operations Management, 430K Olin Hall, University of Southern California, Los Angeles, CA 90089-1453, USA. fudwadia@usc.edu

# Department of Aerospace and Mechanical Engineering, University of Southern California, Los Angeles, CA 90089, USA. mylapill@usc.edu

Abstract

Energy control of an underactuated general nonlinear n-mass lattice (chain) with fixed-fixed and fixed-free boundary conditions is considered. Recent results in analytical dynamics that deal with constrained motion are used to recast the energy control requirement as a constraint on the mechanical system. The fundamental equation of mechanics [1] is used to obtain the explicit nonlinear control force in closed form when a set of k masses (1 ≤ k < n) out of the n masses in the lattice are actuated (see Figure 1 below). Since the number of masses, n, in the lattice could be very large, this paper focuses on the underactuated control of this n degree-of-freedom spatially inhomogeneous, nonlinear system. Sufficient conditions on the locations for placing the actuators are derived using Lasalle’s invariance principle [2] and the underactuated control developed herein is shown to guarantee global asymptotic convergence from any given initial (nonzero) energy state to any desired (nonzero) energy state. In particular, as explained below, such global convergence is guaranteed with the use of just one or two actuators appropriately located for a lattice containing any number of masses.

Figure 1. A general inhomogeneous nonlinear lattice

The paper considers an inhomogeneous, nonlinear lattice that is made up of a chain of masses wherein each mass is connected to its nearest neighbor by a linear or nonlinear memoryless elastic spring element (see Figure 1). The general nature of the lattice is underscored by the fact that (i) the masses in the lattice can all be chosen to be different from one another, (ii) the qualitative nature of each spring element along the lattice can be different, and (iii) the parameters used in the potential functions that describe each spring element can have different values. By qualitative nature of the spring elements, we mean that each of the springs along the lattice can have different restoring-force characteristics, e.g., linear, cubic, quintic, exponential, etc. Thus, the nonlinear lattice that is considered in the present study is strongly inhomogeneous. This is in contrast to the current literature on the subject which almost exclusively deals with either homogeneous lattices (in which the masses are taken to be identical as also the nature and parameter values of the spring elements), or weakly inhomogeneous lattices (in which the inhomogeneity in the lattice is created by the presence of
small, spatially localized perturbations from uniform values [e.g., 3]). The spring force associated with each spring element in the lattice is assumed to be derivable from a potential function that is: (i) twice continuously differentiable, (ii) strictly convex possessing a global minimum at zero displacement, and (iii), has zero curvature possibly only at zero displacement. These assumptions are in conformity with the structural models used to describe many real-life situations. Additionally, as in many materials, asymmetries in the potential functions, which result in differences between tensile and compressive behavior, are included. Such general nonlinear lattices exhibit highly nonlinear behavior and can display intricate and disparate structures such as solitons, phonons, and breathers, in their dynamical response.

The control methodology is developed in three steps. The first step involves writing the equations of motion of the uncontrolled system, which can be easily done using Newton’s laws (see Figure 1). The second step involves the formulation of the equations that characterize the constraints. The constraints fall into two categories. The first is the energy constraint, which deals with our objective to increase/decrease the energy of the lattice from a given initial value, \( H_0 \neq 0 \), to a desired final value, \( H_f \neq 0 \). The second category of constraints are the so-called ‘no-control constraints’. These deal with the fact that we want to consider underactuated systems wherein only \( k \) masses (\( 1 \leq k < n \)) in the lattice out of a total of \( n \) are actuated. The final step of the control methodology deals with the use of analytical dynamics to obtain a closed form expression for the explicit nonlinear control force, \( F^c \). With the uncontrolled equations of motion and the explicit control force at our disposal, the controlled equations of motion of the nonlinear lattice are obtained. The control forces \( F^c \) to be applied at the actuator locations are found to be highly nonlinear and resemble those found in self-excited systems such as Van der Pol-type oscillators. Inspite of the general nature of the nonlinear lattice considered in this study, the control is obtained in closed form without the need to make any approximations or linearizations of the nonlinear dynamical system and without the need to impose any a priori structure on the nature of the controller.

Stability analysis of the control so obtained is performed through the use of LaSalle’s invariance principle [2]. The principle is used to show global asymptotic convergence to the desired energy state and to simultaneously obtain sufficient conditions on the placement of actuators so that such convergence is guaranteed for the underactuated system. Specifically, the control force obtained is shown to provide global asymptotic convergence from any given (nonzero) initial energy state, \( H_0 \), to any desired (nonzero) final energy state, \( H_f \), provided that either (i) the first mass of the lattice, \( m_1 \), or (ii) the last mass, \( m_n \), or alternately, (iii) any two consecutive masses of the lattice, are included in the set of masses that are actuated (see Figure 1).

Several numerical simulations are presented for a 101-mass nonlinear lattice in which only one or two masses are actuated to control the energy from any given initial value to its final desired value. The closed form analytical-dynamics-based control that is obtained is shown to perform well demonstrating the ease, simplicity, and accuracy with which the methodology works.

References


Analytical and numerical control of global bifurcations in a noncontact atomic force microcantilever

Valeria Settimi*, Giuseppe Rega*, Stefano Lenci#

* Department of Structural and Geotechnical Engineering
  Sapienza University of Rome
  Via A. Gramsci 53, 00197 Rome, Italy
  [giuseppe.rega, valeria.settimi]@uniroma1.it

# Department of Civil and Building Engineering, and Architecture
  Polytechnic University of Marche
  Via Brecce Bianche, 60131 Ancona, Italy
  lenci@univpm.it

Abstract

In AFM literature several control techniques have been proposed in the last decades to improve microscope performances or avoid unwanted behaviors. They are mostly based on feedback methods aimed at controlling local dynamics, so their effects on the overall dynamics are generally unknown. Recent studies have shown that the insertion of an external feedback control in a noncontact AFM model causes a generalized reduction of the stability region and a dangerous decrease of system safety in terms of basins of attraction robustness and integrity [4]. Yet, in a practical stability perspective, an acceptable system-dependent residual integrity is needed to guarantee secure AFM operation. Thus, focusing on the preservation of dynamical integrity, a different control technique is applied to the AFM system with the aim to reduce the safe basin erosion which leads to the loss of safety. The method [1] consists of optimally modifying the shape of the excitation to delay the occurrence of the global events (i.e., homo/heteroclinic bifurcations of some saddle) which trigger the erosion (i.e., the sudden fall in the erosion profiles), thus obtaining an overall control of the dynamics and an enlargement of the system safe region in parameters space. Following the procedure, controlling superharmonics are added to the harmonic excitation of the reduced order noncontact AFM model already studied in [3]:

\[ \ddot{x} + \alpha_1 x + \alpha_3 x^3 + \frac{\Gamma_1}{(1 + x)^2} + \rho_1 \dot{x} + \sum_{j=1}^{n} U_j \sin(j \omega t + \Psi_j) \]  

(1)

where \( x \) is the transverse microcantilever displacement; \( \alpha_1 \), \( \alpha_3 \) are linear and cubic stiffness terms; \( \rho_1 \) is the linear damping coefficient; \( \Gamma_1 \) is the attractive atomic interaction coefficient; \( U_j \) is horizontal parametric-like reference excitation while \( U_j \) and \( \Psi_j \) are the amplitudes and phases of the controlling superharmonics. Usually, global bifurcations triggering erosion involve the stable and unstable manifolds of the hilltop saddle. Their control has proved to be effective in delaying erosion also when the fall of the profiles is likely due to other secondary bifurcations which involve internal saddles [2]. Analyses are thus focused on the homoclinic bifurcation of the hilltop saddle, which is analytically detected by the classical Melnikov method. Then, the control method is applied by stating the optimization problem for the best excitation shape, limiting the analyses to the addition of a single controlling superharmonic (\( j = 2 \)). The critical amplitude of the controlled system results \( U_{1,cr}(\omega) = U_{1,cr}^h(\omega)/M(\omega) \), with \( M(\omega) = \max\{ \sum_{j=1}^{2} \frac{U_j l_2(\omega)}{L_{2}(\omega)} \cos(j \omega t_0 + \Psi_j) \} \). Here, \( U_{1,cr}^h(\omega) \) is the critical amplitude for homoclinic bifurcation with the reference harmonic excitation and \( l_2(\omega) \) is an oscillating function exponentially converging to zero for \( t \rightarrow \infty \). Figure 1(a)
shows the positive effect of the added superharmonic on the increase of the homoclinic bifurcation threshold detected via the Melnikov method. To verify also the shift of the erosion process, several basins of attraction of the controlled system are obtained for increasing values of the forcing amplitude around the system fundamental frequency ($\omega \approx 0.8375$), and the relevant erosion profiles are built by means of the Global Integrity Measure GIM and the Integrity Factor IF, respectively accounting for, and getting rid of, the system transient dynamics. The results are compared in Fig. 1(b) with those obtained for the reference system and highlight that, differently from [2], the shift of the hilltop saddle homoclinic bifurcation (HOM SH) does not manage to delay the sharp erosion due to the fact that it occurs too far before the fall down of the erosion profiles, thus vanishing the effect of the control. In fact, accurate numerical investigations of the reference model display the occurrence of other two global bifurcations, i.e., a homoclinic bifurcation of the internal saddle manifolds (HOM S1) and a heteroclinic bifurcation between hilltop and internal saddles manifolds (HET S1-SH). They arise very close to the profile fall down, with the last one being directly related to the erosion triggering. Ongoing studies are thus focused on the application of the control technique to such global event, even if the impossibility of analytical treatment calls for a fully numerical approach both to detect the invariant manifolds and their bifurcations and to verify the effects of control by comparison with the uncontrolled system outcomes.

![Homoclinic bifurcation thresholds and erosion profiles](image.png)

Figure 1: Homoclinic bifurcation thresholds (a) and erosion profiles at $\omega = 0.7$ (b) for the reference (black) and controlled (orange) system with 1 optimal superharmonic ($U_2/U_1 = 0.1347$).

References


Effective Hamiltonians for Resonance Interaction Dynamics and Interdisciplinary Analogies

Valery N. Pilipchuk

Mechanical Engineering
Wayne State University
Detroit MI, 48202, USA
pilipchuk@wayne.edu

Abstract

Resonance interactions of oscillators are responsible for fundamental effects in different areas of physics and classical mechanics. The resonance between any two oscillators/modes destroys their individuality by generating a new effective oscillator of energy flow between the two parent oscillators, which is known as beating. In particular, the fundamental character of such energy exchange oscillators is revealed by the fact of their exact integrability in many physically reasonable cases. The present work illustrates such a standpoint on elastic oscillators and discrete liquid sloshing models. In the case of elastic oscillations, a strongly nonlinear conservative oscillator describing the dynamics of energy partition between two identical linearly coupled oscillators with polynomial restoring force characteristics is analyzed.

The approach is illustrated on the mechanical model whose schematic diagram is shown in Figure 1a, and the differential equations of motion are of the form

\begin{align}
\ddot{q}_1 + 2\zeta\Omega \dot{q}_1 + \Omega^2 q_1 + H_1(q_1, q_2, \dot{q}_1, \dot{q}_2) &= 0 \\
\ddot{q}_2 + 2\zeta\Omega \dot{q}_2 + \Omega^2 q_2 + H_2(q_1, q_2, \dot{q}_1, \dot{q}_2) &= 0
\end{align}

(1)

where $\Omega$ is the frequency of both generating conservative oscillators, $\zeta$ is a damping ratio, $H_1$ and $H_2$ are (nonlinear) polynomials.

We show that a complete dynamic characterization of system (1), except the fast phase component, is provided by the following slowly varying quantities [1]:

\begin{align}
E_{11} + E_{22} &= E_0 & \text{- Total energy excluding coupling} \\
\frac{E_{11} - E_{22}}{E_{11} + E_{22}} &= P = -\sin \theta & \text{- Energy distribution index} \\
\frac{E_{12}}{\sqrt{E_{11}E_{22}}} &= Q = -\cos \Delta & \text{- Coherency of vibrations index}
\end{align}

(2)

where all the values $E_{kj} = (\dot{q}_k \dot{q}_j + \Omega^2 q_k q_j)/2$ ($k = 1, 2; j = 1, 2$) are in energy units; geometrical meaning of the coherency index $Q$ is illustrated in Figure 1b for the case $P = 0.$
Then, we prove that, in many physically meaningful cases of system (1), the combination $\{P, \Delta\}$ represents a set of conjugate Hamiltonian variables described by the differential equations

$$
\begin{align*}
\frac{dP}{ds} &= -\frac{\partial H}{\partial \Delta}, \\
\frac{d\Delta}{ds} &= \frac{\partial H}{\partial P}
\end{align*}
$$

(3)

where $H$ is an effective Hamiltonian, $s$ is a slow temporal scale, which depends on the coefficients of polynomials $H_1, H_2$ and the damping ratio $\zeta$, and $\kappa$ is a single constant parameter of the Hamiltonian that occurs during the derivation process and completely determines the phase plane structure of the energy exchange oscillator. Furthermore, system (3) is exactly solvable in terms of the elliptic functions. However, drawing the level lines of Hamiltonian $H(P, \Delta)$ allows for complete characterizations of all possible dynamics.

We noticed that, despite of very different physical contents and analytical approaches to system’ reduction, the final form of effective Hamiltonian appears to have surprisingly similar mathematical structure, which is known as Boson Josephson Junction in macroscopic quantum dynamics, where equations of the form (3) are obtained within quite different formalism of complex representations [2]. Such a similarity allows us to track interesting physical analogies while revealing a common geometrical nature of resonance modal interactions. For instance, from such standpoint, the so-called nonlinear normal modes of elastic systems and liquid sloshing modes appear to be the same type of phenomena.

**References**


Characterization of dynamic properties of a nonlinear mechanical joint from experimental data

Miguel R. Hernandez-Garcia\textsuperscript{1,2}, Sami F. Masri\textsuperscript{2}, Roger Ghanem\textsuperscript{2}

\textsuperscript{1}Alta Vista Solutions
3260 Blume Dr. Suite 500
Richmond, CA 94806, U.S.A.
mhernandez@altavistasolutions.com

\textsuperscript{2} Viterbi School of Engineering
University of Southern California
Los Angeles, CA 90089, U.S.A.
[masri, ghanem]@usc.edu

Abstract

A formidable hurdle preventing the use of sophisticated computational tools, replicating the behavior of existing physical structures, or simulating the response of complex nonlinear systems, is the lack of high-fidelity, physics-based, robust nonlinear computational models that accurately characterize the behavior of such systems under arbitrary dynamic environments. Related issues that are crucial to the development of useful computational models (which, e.g., can be relied on to reduce the number of physical tests needed for certification of actual physical systems), on the basis of measurements obtained from nonlinear physical models, are (1) the quantification of uncertainties that are inherently present in the underlying physical prototype, (2) the determination of the corresponding uncertainties in the identified reduced-order, reduced-complexity mathematical model, and (3) the subsequent quantification of the propagation effects of these physical uncertainties in the identified model, as well as the uncertainty bounds on the dynamic system response.

This paper presents a study of a general and completely data-driven methodology for characterizing the dynamics of uncertain and complex nonlinear structural components through the development of probabilistic reduced-order models. This proposed methodology, based on the initial work of Ghanem et al. \cite{3}, relies on: (1) a data-driven nonlinear identification approach used to represent the nonlinear non-conservative restoring forces in the system in terms of two-dimensional Chebyshev polynomial expansion \cite{4}, and (2) the polynomial chaos representation of random quantities \cite{2}. Experimental vibration data from a series of dynamic tests performed at Sandia National Laboratories (SNL) on nine nominally identical specimens of a particular type of lap bolted joints with an inclined interface was used to validate the proposed methodology and to develop a probabilistic data-based reduced-order model to characterize the dynamic behavior of this particular type of nonlinear mechanical joint. The probabilistic reduced-order model of the mechanical joint will be defined, in general, as follows:

\[ G(z, \dot{z}) \approx \hat{G}(z, \dot{z}) = \sum_{q=0}^{Q} \sum_{r=0}^{R} C_{qr} T_q(\tilde{z}) T_r(\tilde{\dot{z}}) \]  

(1)

where \( C_{qr} \) are the Chebyshev coefficients, \( T_k(\cdot) \) is the Chebyshev polynomial of order \( k \), and \( \tilde{z} \), \( \tilde{\dot{z}} \) are the normalized displacement and velocity, respectively. This representation of the restoring force, can then be converted into a two-dimensional polynomial expansion of the form

\[ \hat{G}(z, \dot{z}) = \sum_{q=0}^{Q} \sum_{r=0}^{R} a_{qr} z^q \dot{z}^r \]  

(2)

where \( a_{qr} \) are constant coefficients, and \( z, \dot{z} \) are the original displacement and velocity. By relying on Karhunen-Loeve (KL) and polynomial chaos (PC) expansions, the random restoring force

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coefficients $a_{ij}$ can be characterized using a probabilistic reduced representation of the form [1]

$$a \approx \hat{a} = p^0 + \sum_{k=1}^{d} \lambda_k \sum_{|\alpha|=1}^{r} p_{\alpha,k}^0 H_{\alpha}(\xi) \phi_k$$

where the random vector $a$ with values in $\mathbb{R}^m$ comprises all $m$ random coefficients in the reduced-order representation of the restoring force (Eq. 2). The terms $\lambda_k$ and $\phi_k$ correspond to the $d$ most significant eigenvalues and associated eigenvectors of the sample covariance matrix.

Once the probabilistic reduced-order representation of the dynamics of the bolted joint has been constructed from experimental data, it is possible now to quantify the inherent uncertainties in the bolted joint specimens, estimate uncertainty bounds on the dynamic response of the joint, and propagate the effects of these physical uncertainties in the response of a more complex system where bolted joints are considered as critical components. Figure 1 displays the estimated restoring force surface and corresponding $\pm 2\sigma$ bounds for the tested lap-type bolted joint.

**References**


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Simulation and Optimization of the Dynamical-Optical Behavior of Mirror Systems

Johannes Störkle, Nicolai Wengert, Peter Eberhard

Institute of Engineering and Computational Mechanics
University of Stuttgart
Pfaffenwaldring 9, 70569 Stuttgart, Germany
[johannes.stoerkle, nicolai.wengert, peter.eberhard]@itm.uni-stuttgart.de

Abstract

High-performance optics, especially lithographic objectives for EUV or astronomic telescopes, are highly resoluting optical systems consisting of precise mirrors. The mirrors are mounted with high accuracy and they are very sensitive with respect to vibrations. Wafer scanning systems use lithography objectives to project structures in a reticle onto wafers, and the wafers are exposed for a certain time. During that time, even small vibrations of the mirrors can be sufficient to produce aberrated images [2]. Sources of these small vibrations can be minimal excitations at the objective frame, e.g., from noise of coolers or influences of the wafer motion system. The dynamical behavior of the optical systems can be described by rigid body motion and small deformations which also influence the optical behavior. In order to investigate the dynamical-optical behavior, multidisciplinary methods and models are necessary.

Here, methods are proposed for simulating and optimizing the performance of dynamical-optical systems. A generalized workflow for dynamical-optical simulations using cooperating software packages according to Figure 1 can be introduced. On the one hand, existing software-tools for the mechanical simulation can be used and on the other hand, an optical toolbox is developed, where ray tracing and Fourier optical methods are implemented. In order to calculate the optical aberrations of deformed surfaces, the mode shapes can be approximated e.g. using Zernike polynomials. Furthermore, an optical simulation yields the related wavefront aberrations. Since the deformation of a dynamical excited surface results from the superposition of the mode shapes, the corresponding wavefront aberrations can also be superposed. In conclusion, the nonlinear dynamical-optical behavior can be investigated.

As an example, the performance of simulating a flexible lens mounted on an elastic bearing is discussed and the results are presented. Beside lens elements, mirrors can also be simulated and analysed. It is shown how a parabolic mirror with obstruction can be modeled and investigated using FE-Tools, multibody software and the developed optical simulation toolbox. In order to decrease the computational effort, different methods of model reduction are tested and compared within the workflow.

Furthermore, an optimization method is introduced which couples and describes the dynamical and optical behavior by means of a transfer function according to [4]. Instead of using active or passive damping strategies and suppressing any motion, it is proposed to optimize the suspensions by modifying the mass and stiffness distributions while analyzing the optical response. Therefore, the optical sensitivities of the mode shapes can be calculated. A simple exemplary optical system with two mirrors modeled as a rigid multibody system [3] is investigated and optimized [1] within this work.
Figure 1: General workflow of dynamical-optical simulations.

References


On the mathematical analysis of vibrations of axially moving strings

Wim T. van Horssen

Delft Institute of Applied Mathematics
Delft University of Technology
Mekelweg 4, 2628 CD Delft, Netherlands
email: w.t.vanhorssen@tudelft.nl

Abstract

In this paper the transversal vibrations of an axially moving string with constant or time-varying length, time-varying velocity, and time-varying tension are studied. By using a multiple timescales perturbation method asymptotic approximations of the solutions of the formulated initial-boundary value problems are constructed. The applicability of Galerkin's truncation method and the applicability of the method of characteristic coordinates for these types of problems are discussed. The presence of internal resonances and autoresonances are described in detail.

Introduction

Many engineering devices are represented by axially moving continua [1]. In many applications, systems like conveyor belts, elevator cables [2,3], paper sheets, satellite tethers, flexible appendages, crane and mine hoists [4], and cable-drive robots exhibit constant or variable-length, and transport speed during operation. To improve the design of elevators, one of the major tasks is to develop a better understanding of elevator cable dynamics and new methods to effectively reduce the vibration and noise. The dynamics of media with variable-length, velocity and tension is the subject of this paper. Due to small allowable vibrations the lateral and vertical cable vibrations in elevators can be assumed to be uncoupled and only lateral vibrations are considered here. The elevator car is modelled as a rigid body of mass \( m \) attached at the lower end of the cable, and the suspension of the car against the guide rails is assumed to be rigid, where the external excitation due to wind is also considered at the boundary. This is considered to be a basic and a simple, asymptotic string model of an elevator cable from practical viewpoint. The initial boundary value problems will be studied, and explicit asymptotic approximations of the solutions, which are valid on a long timescale, will be constructed. Two cases for varying-length are considered, (i) \( l(t) = l_0 + vt \) with boundary excitations \( \nu(O,t) = \alpha \sin(\Omega t) \), where \( l_0 \) is the initial cable length, \( v \) denotes the constant cable velocity, \( \alpha \) is the oscillation amplitude of the building from equilibrium and \( \Omega \) is the frequency of this excitation and (ii) \( l(t) = l_0 + \beta \sin(\omega t) \), where \( \beta \) defines a length variation parameter and \( \omega \) signifies the angular frequency of length variation and \( l_0 > |\beta| \). For the harmonic length-variations, it will be shown that Galerkin's truncation method cannot be applied in order to obtain asymptotic results on long timescales, and for boundary excitations and linear length variations, the interesting phenomena of autoresonance, see for instance [5,6,7] will be discussed in detail. This phenomena arise when there occurs a passage through dynamic resonance. It will also be shown that an \( O(\varepsilon) \)-amplitude excitation gives rise to \( O(\sqrt{\varepsilon}) \) responses. For further details we refer the reader to [6,7]. For conveyor belt problems it will be shown how the two timescales perturbation method in combination with the method of
characteristic coordinates can be used to construct asymptotic approximations of the solutions on long timescales. Also for these conveyor belt problems it turned out that Galerkin’s truncation method was not applicable to obtain asymptotic results on long timescales.

Mathematical Models
Relative to the fixed coordinate system the lateral displacement of the string particle instantaneously located at spatial position \( x \) at time \( t \), where \( 0 \leq x \leq l(t) \), is described by \( u(x,t) \). The equations of motion for a vertically moving string with time-varying length, velocity and tension are formulated and are given by

\[
\rho \left( \frac{u_{xx} + 2u_{x} + v u_{x} + v^2 u_{xx}}{P(x,t)} \right) = \left( P(x,t) u_{x} \right)_t = 0, \quad t > 0, \quad 0 < x < l(t),
\]

\[
u(0,t) = 0 \quad \text{or} \quad \alpha \sin(\Omega t), \quad u(l(t),t) = 0, \quad t > 0,
\]

\[
u(x,0) = f(x), \quad \text{and} \quad u_t(x,0) = h(x), \quad 0 < x < l(0),
\]

where the subscript for \( u \) denotes partial differentiation, \( P(x,t) = mg + \rho(l(t) - x)g - mv^2 - \rho(l(t) - x)v^2 \) is an axial force arising from its own weight and longitudinal acceleration, \( \alpha \) is the excitation amplitude at the upper end, \( \Omega \) is the excitation frequency, \( g \) is the acceleration due to gravity, and where \( f(x) \) and \( h(x) \) represent the initial displacement and the initial velocity, respectively.

Conclusions
A multiple timescales perturbation method has been used in search of infinite mode approximate solutions. It is observed that there are applicability issues with Galerkin’s truncation method for string-like problems. A set of new problems have arisen in studying the elevator cable system under weak boundary excitations which is related to the investigation of a physical phenomenon known as autoresonance.

References:
Nonlinear free vibrations of planar elastic beams: A unified treatment of geometrical and mechanical effects

Stefano Lenci*, Giuseppe Rega#

* Department of Civil and Buildings Engineering, and Architecture Polytechnic University of Marche Via Brecce Bianche, I-60131 Ancona, Italy lenci@univpm.it

# Department of Structural and Geotechnical Engineering Sapienza University of Rome via A. Gramsci, I-00197 Rome, Italy giuseppe.rega@uniroma1.it

Abstract

Nonlinear free vibrations of planar elastic beams have been the subject of many literature studies aimed at investigating the effects of kinematical and dynamical assumptions under different boundary conditions [1-3]. Yet, open issues still occur as regards the influence of the various assumptions (concerned with axial inertia, rotational inertia, shear deformability, and nonlinear curvature) in variable geometrical situations. This paper aims at comparatively highlighting their importance via a unified treatment of the nonlinear free vibrations of hinged beams with either an axially restrained ($\kappa \to \infty$, hinged-hinged) or free ($\kappa = 0$, hinged-supported) boundary (see Fig. 1, where $\kappa$ is the stiffness of the end spring).

The beam model is developed directly in a 1D framework by considering linear elastic behavior, since the paper focus is on the effects of kinematical and dynamical assumptions. Three geometrical exact equations of free motion in the axial and transverse displacement and in the shear angle are obtained. They are strongly nonlinear and cannot be solved in closed form, so the Lindstedt-Poincaré method is applied to obtain an analytical solution, which is then investigated in detail. Longitudinal and transverse dynamics are decoupled at first order, which allows to assume $W_1 = 0$ and obtain the equation of motion in the sole transverse displacement

$$EJ \frac{\partial^4 U_1}{\partial Z^4} - \left( EJ \frac{\rho A}{GA} + \rho J \right) \omega_0^2 \frac{\partial^4 U_1}{\partial Z^2 \partial t^2} + \rho A \omega_0^2 \frac{\partial^2 U_1}{\partial t^2} + \frac{\rho J \rho A}{GA} \omega_0^4 \frac{\partial^4 U_1}{\partial t^4} = 0. \quad (1)$$

It provides the first order normal mode and the linear natural frequencies $\omega_0$ of the system, both of which do not depend on the end-spring stiffness $\kappa$. The second order problem provides the axial displacement $W_2(Z,t) = W_{2a}(Z) + W_{2b}(Z) \cos(2t)$, which depends on $\kappa$. The nonlinear frequency correction $\omega_2$ is provided by the solvability conditions of the third order problem, and depends on the square of the oscillation amplitude $U_a$:

$$\omega = \frac{1}{L^2} \sqrt{ \frac{EJ}{\rho A} \left( \frac{\varpi_0}{L} + \left( \frac{\varpi U_a}{L} \right)^2 \right) \varpi_2 + \cdots } . \quad (2)$$

The main goal consists of studying the dependence of $\varpi_2$ on the axial and rotational inertia, on the shear stiffness and, mainly, on the kind of axial (immovable or movable) boundary. The accomplished comprehensive treatment allows us to resume and properly framework literature.
results, while also shedding new light on the comparative importance of the beam geometrical and mechanical properties.

For hinged-hinged beams, Fig. 2 shows the function \( \bar{\omega}_{2}(l) \) (\( l = L/\sqrt{A/\bar{I}} \) is the slenderness) without (a) and with (b) rotational inertia being considered, \( z \) representing the dimensionless stiffness \( (z \to \infty \) for Euler-Bernoulli beams) and the two curves for each \( z \) value corresponding to vanishing or non-null axial inertia. The shear stiffness is seen to play a major role, the most important result being the transition of the beam nonlinear behavior from hardening \( (\bar{\omega}_2 > 0) \) to softening \( (\bar{\omega}_2 < 0) \) which occurs in the low (but still in the realm of practical applications) range of slenderness in the case of (nearly) shear indeformability. Rotational inertia has no meaningful influence for low \( z \) values, but strongly affects the nonlinear behavior of shear indeformable beams \( (z = \infty) \) in the low slenderness range, where neglecting it along with the shear deformability (as it is often made in the literature) entails a definitely stronger softening behavior. The importance of shear stiffness and rotational inertia mostly for low slenderness values is confirmed when considering hinged-supported beams.

In contrast, the role of axial inertia meaningfully depends on the kind of axial boundary condition. For hinged-hinged beams, it does not affect the results (Fig. 2), thus validating the ‘static condensation’ procedure usually accomplished in the literature to express the axial displacement in terms of the transverse one. Instead, for hinged-supported beams, axial inertia plays a major role in the nonlinear free vibrations, which consists of the qualitative change from hardening to softening of the asymptotic value attained by the nonlinear frequency correction for increasing slenderness, along with nontrivial modifications in the medium-low range of slenderness values.

![Figure 2. The function \( \bar{\omega}_2(l) \) without (a) and (b) with rotational inertia. Hinged-hinged beam.](image)

Ongoing investigations aim at drawing possibly comprehensive conclusions as to the geometrical and boundary situations in which either one of the various beam mechanical properties can be reliably disregarded in the analysis of nonlinear free vibrations.

References


On the averaging in strongly damped systems: The general approach and its application to asymptotic analysis of the Sommerfeld effect

Alexander Fidlin*, Olga Drozdetskaya*

* Institute for Engineering Mechanics, Karlsruhe University of Technology, Kaiserstraße 10, D- 76131 Karlsruhe, Germany
alexander.fidlin@kit.edu, olga.drozdetskaya@kit.edu

Abstract

Considering asymptotic procedures for nonlinear dynamics, the focus is usually on systems which are close to integrable ones [1]. However, systems with complete or partially strong damping are usual in applications, especially in control systems. The strong recent trend to light weight design makes mechanical structures extremely sensitive against any kind of vibration excitation. Thus tightly focused damping has to be applied in order to reduce vibration amplitudes to a certain acceptable level without decreasing the efficiency of a mechanism as a whole significantly.

A very simple approach for asymptotic analysis of partially damped dynamic systems has been suggested in [2]. This method is generalized in the present paper for the case of the damped part of the system depending strongly on the weakly damped slow variables. Subsequently the described approach is applied for analysing of the passage through resonance in a strongly damped system excited by an induction motor of limited power.

Consider the following system:

\[ \dot{x} = \epsilon X(x, y, t), \quad \dot{y} = -k(x) y + \epsilon Y(x, y, t), \]
\[ x \in D \subset \mathbb{R}^n, \quad k(x) \geq k_0 > 0 \quad \text{for all} \quad x \in D, \quad \epsilon << 1. \]

Alongside with (1) consider the averaged equations

\[ \ddot{\xi} = \epsilon \Xi(\xi); \quad \eta = -k(\xi) \eta; \quad \Xi = \lim_{T \to \infty} \frac{1}{T} \int_0^T X(\xi, 0, t) dt. \]

Under certain assumptions on the smoothness of the functions \( k, X, Y \) it can be shown that the solutions of these systems corresponding to the same initial conditions are close to each other on the asymptotically long time interval \( 0 \leq t < O(1/\epsilon) \).

This approach can be applied for analyzing the passage through the resonance in a strongly damped oscillating system interacting with an inertial exciter connected to an induction motor of limited power (cf. Fig. 1). The equations of motion for this system can be written as follows:

\[ L(\dot{\phi}) = mL(\dot{\phi}) + m\epsilon \dot{\phi} \sin \phi; \quad (M + m) \ddot{x} + \beta \dot{x} + cx = m\epsilon \dot{\phi} \cos \phi + \dot{\phi} \sin \phi \]

Introducing appropriate non-dimensional parameters and linearizing the torque characteristics of the induction motor these equation can be transformed to the basic form for the further analysis:

\[ \phi'' = u(\Omega - \phi') + \epsilon \dot{\phi} \sin \phi, \quad z'' + 2Dz' + z = \phi^s \cos \phi + \phi^s \sin \phi, \]
\[ x = \frac{m}{M + m} \epsilon z, \quad k = \sqrt{\epsilon \frac{c}{M + m}}, \quad \tau = k t, \quad (\dot{\phi})' = \frac{d}{d\tau}; \quad D = \beta / 2, \quad (M + m) I \ll 1. \]

Here the parameters \( \epsilon \) and \( u \) are assumed to be small, similar to [3]. However the damping \( D \) is not assumed to be small, which simplifies both the analysis and the dynamical behaviour of the system during passage through the resonance significantly [4].
The simplest oscillating system interacting with the inertial exciter of limited power (a) and the torque characteristics of the induction motor (b)

The averaged dynamics of the system can be described by an equation of the first order:

\[
\frac{d\omega}{d\varphi} = \frac{1}{\omega} \left( u(\Omega - \omega) - V(\omega) \right), \quad V(\omega) = \frac{eD^2\omega^2}{\left(1 - \omega^2\right)^2 + 4D^2\omega^2}.
\]

This equation determines first of all the stationary rotation speeds of the exciter and their stability which is illustrated in Figure 2. On the other hand, it enables the development of efficient strategies for avoiding the Sommerfeld effect [3] by applying appropriate passive (manipulating \(D\)) or active (manipulating \(u\)) control.

Figure 1. The simplest oscillating system interacting with the inertial exciter of limited power (a) and the torque characteristics of the induction motor (b)

Figure 2. Stationary regimes of the system and their stability

References


Attractivity and Bifurcations of Stationary Solutions in Systems with Non-Smooth Frictional Damping

Hartmut Hetzler
University of Kassel
Department of Mechanical Engineering
Institute for Mechanics, Applied Dynamics Group
Mönchebergstrasse 7, 34125 Kassel, Germany
hetzler@uni-kassel.de

Abstract

Stability problems are usually analyzed assuming smooth damping forces. However, in practical applications often dissipation stemming from micro-slip in joints or interfaces is observed which gives rise to non-smooth frictional damping forces of finite intensity (cf. [4],[9],[1] for instance). While the influence of joint damping on forced vibrations has been extensively investigated (e.g. [2], [8], [4]), investigations on the effects of non-smooth damping forces on stability problems are surprisingly rare (e.g. [10],[11], [12] and recently [5],[6], [7]). Within this contribution the impact of non-smooth joint damping on different instability mechanisms is investigated with regard to attractivity and bifurcation of equilibrium sets and limit-cycles.

Joint forces are often modeled using combinations of rheological models involving Coulomb elements. Applying such force models, the resulting equations of motion may be written as

$$\dot{q} = v, \quad \dot{v} = -[D + G]v - [K + N]q - f_{NL} \in -J^\top R \text{Sign}(Jv)$$

(1)

where $f_{NL}$ collects all smooth nonlinear terms of higher order, $R = \text{diag}(\rho_i)$ contains the intensity of joint friction and $Jv = v_{rel}$ is the relative velocity in the interface. $\text{Sign}(u) = \overline{\text{co}}(\text{sign}(u))$ indicates the convex closure of the classical signum function $\text{sign}(u)$ and applies element-wise to matrix arguments. Since equation (1) is of Filippov type, existence of solutions is guaranteed [3]. Moreover, it can be shown that near switching hyperplanes $v_i = 0$ the flow of (1) gives either rise to transversal crossings or attractive sliding modes. Thus, solutions are unique in forward time [7].

Equilibrium solutions to (1) are given by the set

$$\mathcal{E} = \left\{ (q_0, v = 0) \mid 0 = [K + N]q_0 - f_{NL}(q_0) + J^\top R \text{Sign}(0) \right\}.$$  

(2)

In a first step, the attractiveness of $\mathcal{E}$ given by (2) for $f_{NL} = 0$ is discussed depending on the properties of the system matrices. In order to handle the non-smoothness of the problem, Lyapunov’s direct method is applied and attractiveness or instability of the equilibrium set is assessed by applying LaSalle’s Invariance Principle or Chetayev’s Instability Theorem. The results are compared to the well known classical findings for the underlying M-DG-KN systems. As a first result it is obvious from (2) that the smooth problem with $R = 0$ will have unique equilibrium points while for $R \neq 0$ equilibria will be convex sets.

Based on equation (1) general results on the properties of equilibrium sets are obtained: For static stability problems, which are controlled by the definiteness of $K$, it is shown that the behavior is not altered fundamentally: adding frictional damping will turn asymptotically stable equilibrium points in attractive equilibrium sets, while unstable equilibria are turned into unstable sets of
equilibria. Thus, the basic results regarding the asymptotic behavior of solutions may simply be transferred to the non-smooth problem. While static stability problems are not too much affected, it is shown that the behavior of kinetic stability problems is altered fundamentally. In particular it is found that adding non-smooth joint-damping to the smooth problem may turn unstable equilibria into attractive sets of equilibria with a finite basin of attraction. It is shown that this stabilizing effect depends on $J$ and corresponding conditions on $J$ will be presented.

Beyond attractivity of equilibria also the behavior of limit cycles is investigated. To this end non-smooth averaging and Galerkin’s method are applied in order to approximate the limit cycles and derive information about their attractivity.

To demonstrate the findings for equilibria and investigate the behavior of limit cycles, a variety of classical static and kinetic stability problems under the additional influence of non-smooth frictional damping is discussed. Beyond the results for the equilibria it is found that joint damping may give rise to additional limit cycles which may particularly alter the bifurcation behavior at small amplitudes. As a consequence, the basins of attraction of attractive solutions may be finite, which may let finite perturbations drive the system from one attractor to another. On the other hand, if carefully designed frictional damping can be useful to control and limit the amplitudes of self-excited vibrations.

References


Enforcing Force-Displacement Proportionality in Nonlinear Systems through the Addition of Nonlinearity

Giuseppe Habib, Gaetan Kerschen

Department of Aerospace and Mechanical Engineering
University of Liège, Belgium
[giuseppe.habib,g.kerschen]@ulg.ac.be

Abstract

Linear systems obey the principle of superposition, i.e., they are characterized by a proportionality between their response and the applied force. Because of the scaling that exists between different forcing levels, engineering design is therefore greatly facilitated. However, nonlinearity is a frequent occurrence in engineering structures and is such that the superposition principle can generally no longer be applied.

The objective of the present study is to introduce nonlinearity into an already nonlinear system to retrieve certain dynamical properties exhibited by linear systems, hence creating a sort of compensation effect. For instance, reference [1] enforced isochronicity, i.e., amplitude-independent resonant frequency, in nonlinear systems. The focus herein is to investigate how the proportionality between the response of the system and the applied force can be retrieved for a specific vibration mode in a large range of forcing levels.

Considering a general mechanical system, described by the system of differential equations

\[ M\ddot{x} + C\dot{x} + Kx + b_{nl} = f_0 \cos(\omega t)f, \]

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices, respectively, \( b_{nl} \) includes the nonlinear terms, \( f_0 \) is the forcing amplitude, \( \omega \) is the excitation frequency, \( t \) is time and \( f \) is a constant vector that locates the applied force. For simplicity, we consider that the system has a single nonlinear element described by a third-order term, \( b_{nl} = \alpha_3 f_0^2 \), where \( \alpha_3 \) is a real parameter.

Normalizing the system using \( y = x/f_0 \), the forcing amplitude and the nonlinearity are expressed by the unique parameter \( \alpha_3 = \alpha_3 f_0^2 \). Applying the harmonic balance method, Eq. (1) can be expressed as a system of nonlinear algebraic equations,

\[ A(\omega)q + \alpha_3 d(q) = c(\omega), \]

where \( q \) collects the amplitude of the different harmonics of the solution, \( d \) contains the nonlinear terms and \( c \) is related to the external forcing.

We introduce in the system another third-order nonlinearity, \( \beta_3 = b_3 \alpha_3 \). Thus, the system of algebraic equations becomes \( Aq + \beta_3 (d + b_3 d_\beta) = c \). Expanding \( q \) with respect to \( \alpha_3 \) as \( q = q_0 + \alpha_3 q_1 + O(\alpha_3^2) \), the approximated frequency response of the system can be obtained explicitly, i.e., \( h = h_0 + \alpha_3 (h_{10} + b_3 h_{13}) + O(\alpha_3^2) \), where \( h_0, h_{10} \) and \( h_{13} \) depend on \( \omega \). Focusing on the frequency response of the first coordinate, we call \( h = h(1) \) and analogously \( h_0, h_{10} \) and \( h_{13} \) for \( h_0, h_{10} \) and \( h_{13} \).

We call \( \alpha_0 \) the resonant frequency of the considered mode of the underlying linear system, and \( \bar{\omega}_0 \) the corresponding resonant frequency when the two nonlinearities are considered. Approximating the difference between \( \alpha_0 \) and \( \alpha_0 \) with a linear function proportional to \( \alpha_3 \), the value of \( b_3 \) that satisfies, in first approximation, the proportionality relation expressed through the objective function
\[ H = -h_0(\dot{\omega}_0) + h(\omega_0) = 0 \]

\[ b_3 = \frac{h_{10}\omega - h_{100}h_{1\omega}}{h_{10}\omega - h_{100}h_{1\omega}} \bigg|_{\omega = \bar{\omega}_0}, \]  

(2)

where the subscript \( \bar{\omega} \) indicates derivation with respect to \( \omega \).

The developments were validated using a two-degree-of-freedom (2DOF) system possessing a cubic spring:

\[
m_1\ddot{x}_1 + k_1 x_1 + c_2 (\dot{x}_1 - \dot{x}_2) + k_2 (x_1 - x_2) + \tilde{\alpha}_3 x_1^3 = f_0 \cos \omega t, \\
m_2\ddot{x}_2 + c_2 (\dot{x}_2 - \dot{x}_1) + k_2 (x_2 - x_1) = 0.
\]

(3)

Fig. 1(a) illustrates that the original nonlinear system is characterized by a strong dependence of the resonance peak amplitude to forcing level. An additional cubic spring with coefficient given by (2) was then introduced in the system between masses \( m_1 \) and \( m_2 \). Fig. 1(b) shows that the force-displacement proportionality can be (almost) maintained in a certain range of forcing amplitudes.

In order to extend the range of displacement-force proportionality, the procedure was generalized to nonlinearities possessing additional higher-order terms. For instance, Fig. 1(c) depicts the improvement that can be obtained when a quintic spring is added in parallel to the cubic spring. A resonance peak that exhibits no visible dependence on forcing amplitude is obtained for variations of the resonant frequency up to 20%: this linear-like regime is then followed by a sudden detuning.

![Figure 1: Normalized frequency response of a 2DOF system for different values of the forcing amplitude. (a) Original nonlinear system; (b) system with an added cubic nonlinearity; (c) system with added cubic and quintic nonlinearities. Dashed line: underlying linear system.](image)

Devices that work properly for linear systems, but fail if the system is nonlinear, represent a possible application of this procedure. This is for instance the case of the nonlinear tuned vibration absorber proposed in [2] that can mitigate a nonlinear resonance in a large range of forcing amplitudes.

References


Dynamics of forced system with vibro-impact energy sink.

Oleg Gendelman, Aviv Alloni

Faculty of Mechanical Engineering
Technion – Israel Institute of Technology,
Haifa, 32000, Israel
ovgend@tx.technion.ac.il

Abstract

We consider a forced response of primary linear oscillator with vibro-impact energy sink. This system exhibits some features of dynamics, which resemble forced systems with other types of nonlinear energy sinks, such as steady-state and strongly modulated responses. However, the differences are crucial: in the system with vibro-impact sink the strongly modulated response consists of randomly distributed periods of resonant and non-resonant motion. This salient feature allows us to identify this type of dynamic behavior as chaotic strongly modulated response (CSMR). It is demonstrated, that the CSMR exists due to special structure of a slow invariant manifold (SIM), which is derived from a multiple-scale analysis of the system. In the considered system, this manifold has only one stable and one unstable branch. This feature defines new class of universality for the nonlinear energy sinks. Very different physical system with topologically similar SIM – the oscillator with rotational energy sink – also exhibits CSMRs. In the system with vibro-impact sink, such responses are observed even for very low level of the external forcing. This feature makes such system viable for possible energy harvesting applications.

The model comprises primary forced-damped linear oscillator with the vibro-impact energy sink (VI NES). The latter in this model is just a straight cavity in the primary mass, in which the impacting particle is allowed to move freely. Sketch of the system is presented in Figure 1. The restitution coefficient $\kappa < 1$. This is the only source of damping in the NES.

Without affecting the generality, we obtain non-dimensional model, as the primary mass and the rigidity of the linear spring are adopted to be unit, and length of the cavity, in which the small mass moves, is adopted to be equal to 2 (see Fig.1).

Figure 1. Sketch of the dynamical system: primary forced linear oscillator with the VI NES

In Figures 2a,b we present the numeric verification for the CSMR for the case of relatively small excitation amplitude, $A=0.1$. Exact values of all simulation parameters are presented in the figure captions.
Figure 2. Chaotic strongly modulated response for the case of relatively small forcing amplitude, $\varepsilon = 0.05, \sigma = 1, \xi = 0.2, k = 0.729, A = 0.1$. (a) Displacement of the primary system; (b) Relative displacement of the NES.

From Figure 2 one can observe that the primary oscillator passes through relatively long periods of steady excitation with short “dischargings” when in resonance with the NES. To confirm this conclusion, we present in Figure 3 the results of wavelet analysis for time series presented in Fig. 2.

Figure 3. Wavelet decomposition of the time series for CSMR, $\varepsilon = 0.05, \sigma = 1, \xi = 0.2, k = 0.729, A = 0.1$. (a) Displacement of the primary system; (b) Relative displacement of the NES.

We see that indeed one can identify “jumps” out of the resonance of the primary oscillator. They are correlated with loss of frequency 1 also for the vibro-impact element. These findings in general conform to the qualitative CSMR scenario described above.
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Abstract

The resonant dynamic behavior of a nonlinear Vibration Absorber (VA) attached to a damped nonlinear structure is investigated analytically via asymptotics and numerically via path following. The response of the reduced-order model, whereby the dynamics of the primary structure are projected onto the mode to control, is evaluated using the method of multiple scales up to the first nonlinear order beyond the resonance [1, 2]. Various mechanical devices have been proposed as vibration absorbers such as hysteretic VAs [3] based on steel wire ropes governed by the Bouc-Wen hysteresis law. In a more recent work [4] a different mechanism was proposed to obtain pinched hysteretic restoring forces that entail a significant enhancement of the VA performance. Here, the asymptotic response of the two-dof system with a one–to–one internal resonance derived by the method of multiple scales up to the second nonlinear order is shown to be in very close agreement with the results of path following analyses. The asymptotic solution lends itself to a versatile optimization scheme by which different optimal tuning criteria for the vibration absorber can be found in semi-closed form.

The following nondimensional form of the equations of motion is considered:

\[
\ddot{x}_1 + 2\xi_1 \dot{x}_1 + x_1 + \beta_1 x_3^3 + \mu (\ddot{x}_1 + \ddot{x}_2) = f \cos \Omega t
\]

\[
\mu (\ddot{x}_1 + \ddot{x}_2) + 2\xi_2 \mu \alpha \dot{x}_2 + \mu \alpha^2 x_2 + \beta_2 x_2^3 = 0
\]

(1)

where \(x_1\) denotes the generalized coordinate by which the motion of the primary structure is parametrized and \(x_2\) is the relative motion of the VA. The system behavior is governed by the nondimensional parameters \((\xi, \mu, \alpha, \beta)\) where \(\xi\) denotes the damping factor of the main structure \((i = 1)\) and of the VA \((i = 2)\), \(\mu = m_2/m_1\) is the mass ratio between the VA mass and that of the primary system, \(\alpha = \omega_2/\omega_1\) is the frequency ratio with \(\omega_i = \sqrt{k_i/m_i}\), \(\beta\) represents the cubic stiffness coefficient of the main structure \((i = 1)\) and of the VA \((i = 2)\). A direct harmonic excitation is applied on the main mass having \(f\) as amplitude and \(\Omega\) as frequency.

By letting \(\mathbf{x} = \Phi \mathbf{q}\), where \(\Phi\) is the modal matrix (having the two eigenvectors as columns), \(\mathbf{q} = [q_1, q_2]^{\top}\) collects the normal coordinates and \(\mathbf{p} = \dot{\mathbf{q}}\) the generalized velocities, projection of the equations of motion into modal space [2] yields

\[
\dot{\mathbf{q}} - \mathbf{p} = 0, \quad \dot{\mathbf{p}} + \mathbf{D}\mathbf{p} + \Lambda\mathbf{q} + \mathbf{f}_u - \mathbf{f} = 0.
\]

(2)

A small positive parameter \(\varepsilon\) is introduced to express the state-space coordinates in series of \(\varepsilon\)

\[
\mathbf{q}(t; \varepsilon) = \sum_{k=0}^{\infty} \varepsilon^{2k+1} \mathbf{q}_{2k+1}(T_0, T_2, T_4) \quad \text{and} \quad \mathbf{p}(t; \varepsilon) = \sum_{k=0}^{\infty} \varepsilon^{2k+1} \mathbf{p}_{2k+1}(T_0, T_2, T_4)
\]

(3)

where \(T_0 = t\) represents the fast time scale and \(T_2 = \varepsilon^2 t\) and \(T_4 = \varepsilon^4 t\) denote the slow time scales. By substituting (3) into (2) and collecting terms of like powers of \(\varepsilon\), one obtains a hierarchy of
linear problems. The first order problem governs the linear free vibrations. The solutions of the higher order problems are obtained enforcing their solvability as the orthogonality of the third and fifth order inhomogeneous terms to the solutions of the adjoint homogeneous problem. These yield the modulation equations for the amplitudes and phases which concur to define the closed-form approximate solution. Figure 1 shows the frequency-response curves of the primary system for the case of optimal tuning and away from optimality in the sense of the equal-peak method due to Den Hartog. The left part illustrates the curves obtained for the case of optimal tuning at different orders of approximation, showing that while the third order solution is not accurate enough, the fifth order solution captures very accurately the system response. The right part of Fig. 1 shows the curves obtained with different values of $\beta_2$ varied about its optimal value $\beta_{2,\text{opt}}$ [5].

![Figure 1: Frequency-response curves of the primary system with different tuning conditions of the nonlinear stiffness parameter of the absorber.](image)

Conclusions

A higher order approximation of the one-to-one internal resonance condition which takes place between the primary structure and a nonlinear vibration absorber was shown to describe very accurately the system dynamics. This asymptotic approach is being embedded into a nonlinear optimization procedure that yields different tuning criteria for the absorber in semi-closed form, a problem often addressed by purely numerical methods. Its extension to absorbers with hysteretic nonlinearities is being investigated and will be detailed in the full manuscript.

References


Extension of the Method of Direct Separation of Motions for Problems of Oscillating Action on Dynamical Systems

Iliya I. Blekhman*, Vladislav S. Sorokin†,‡

* Institute of Problems in Mechanical Engineering RAS  # Mekhanobr-Tekhnika Research & Engineering Corp.
V.O., Bolshoj pr. 61, St.Petersburg, 199178, Russia
V.O., 22 linia, 3, St.Petersburg, 199106, Russia
Iliya.i.blekhman@gmail.com yakimova_ks@npk-mt.spb.ru

† Department of Mechanical Engineering
Technical University of Denmark
Nils Koppels Allé, Building 404, 2800 Kgs. Lyngby, Denmark
vladsor@mek.dtu.dk

Abstract

Oscillating action on nonlinear dynamical systems gives rise to several unusual and sometimes paradoxical phenomena [1]. These phenomena in some cases can be employed to improve existing technological processes and machines, in others, in contrary, lead to accidents and even catastrophes. Such phenomena arising in the field of mechanics are relatively well studied, in particular by means of the general approach, named Vibrational Mechanics [1], and the corresponding analytical method, the Method of Direct Separation of Motions (MDSM).

The main aim of the present work is to extend this general approach for studying dynamical systems from various fields of science, e.g. physics, chemistry, biology and others. Another aim is to present a modification of the approach applicable to study cases when frequency of oscillating action is not much higher than natural frequency (frequencies) of the system considered. To illustrate the approach several relevant problems are examined.

The essence of the approach can be presented in the following way. Let dynamics of a process be described by relation:

\[ Z(x, a, t) = 0, \]  

where \( x \) is state vector of the system considered, \( a \) vector of parameters, \( t \) time, and \( Z \) operator that may represent finite, differential, integral and other equations. In presence of fast oscillating action this relation takes the form:

\[ Z(x + \psi_a(t, \omega \tau), a + \psi_s(t, \omega \tau), t) = F(t, \omega \tau), \]  

where \( \psi_s, \psi_a \) and \( F \) are some functions periodic in the “fast time” \( \tau = \omega t \), and \( \omega >> 1 \) (Notions “fast”, “slow”, “high-frequency” can be formalized, see e.g. [1]).

Practically in all cases change of the vector \( x(t) \) under high frequency action can be represented as

\[ x(t) = X(t) + \psi(t, \omega \tau), \]  

where \( X \) is slow, and \( \psi \) is fast 2\( \pi \)-periodic in time \( \tau \) variable, with average zero. And variable \( X \) is of primary interest.

Under certain assumptions implied in the MDSM it is possible to obtain equation

\[ Z'(X, a^*, t) = 0. \]
Equation (4) of slow motions can be named as \textit{Vibro-transformed Dynamics Equation} (VDE); earlier it was called “equation of oscillatory strobodynamics” [2]).

The work also concerns the modified version of the approach. The corresponding method, named in [4,5] the \textit{modified MDSM}, does not require frequency of oscillating action to be high \( \omega \gg 1 \), so that cases of low-frequency, near-resonant, and also non-resonant excitations can be captured. However, a certain restriction on the sought solutions is imposed.

The method implies consideration of dimensionless equations, and vector \( \mathbf{x}(t) \) to be represented as

\[
\mathbf{x} = \mathbf{X}(T_1) + \mathbf{\psi}(T_1, T_0),
\]

where new independent timescales \( T_0 = \tau = \omega t \) and \( T_1 = \varepsilon T_0 \) are introduced, and \( \varepsilon \ll 1 \) is an artificial small parameter, and variables \( \mathbf{X} \) and \( \mathbf{\psi} \) have the same meaning as above.

As is seen from (5), the restriction on the sought solutions, implied in the modified MDSM, is that only those solutions which are close to periodic, i.e. describe \textit{oscillations with slowly varying amplitudes}, can be determined by the means of the method. And the introduced artificial small parameter \( \varepsilon \) defines proximity of the solution to pure periodic one, i.e. how slow the amplitudes are varying.

Introducing small parameter \( \varepsilon \) enables to employ the modified MDSM for solving problems in which it is impossible to assign such parameter. In particular, strongly nonlinear problems can be considered. It should be noted, however, that the modified MDSM implies several conventional simplifications of the method to be abandoned.

To illustrate the conventional and modified approaches several problems of mechanics, physics, chemistry and biology are considered. In particular, problems of parametric excitation and amplification of oscillations, which recently attract much attention [3], are discussed.

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\textbf{References}


A perturbation scheme to solve a bi-stable energy harvester

Angelo Luongo*, Sara Casciati#, Daniele Zulli*

* M&MoCS International Research Center on Mathematics and Mechanics of Complex Systems
  University of L’Aquila
  via G. Gronchi, 67100 L’Aquila, Italy
  [angelo.luongo, daniele.zulli]@univaq.it

# DICAr-Dipartimento di Ingegneria Civile e Architettura, University of Catania
  Piazza Federico di Svevia, 96100 Siracusa, Italy
  sacasci@unict.it

Abstract

The objective of vibration energy harvesting is to efficiently transform kinetic energy in electricity, in order to power up some kind of electronic devices. A survey on techniques and materials used in this field is given in [7]. Generally, it turns out that linear oscillators, when used as harvesters, are well suited just for stationary and narrow band excitation close to their resonant frequencies, whereas they show less efficiency when an ambient vibrational energy is distributed over a wide spectrum, and dominant at low frequencies. To overcome this drawback, different classes of non-linear oscillators can be used as harvesters, such as nonlinear airfoils under wind effect [1], or bi-stable single d.o.f. systems subjected to slow excitation frequencies below resonance [3]. In particular, the bistable oscillators are characterized by a double-well restoring force potential, and they can be excited at frequencies much less than the linear natural one, being the interwell escape exploited as a frequency up-conversion technique to achieve broadband energy harvesting. In fact, the dynamical responses are separated into two distinct components, related to low and high frequencies (slow and fast oscillations), respectively, where the slow variable modulates the fast one. It gives rise to the specific phenomenon of bursting, consisting in a series of spikes followed by temporary latency, which can be observed in generalized Duffing [5, 4] as well as generalized Van der Pol [6] oscillators.

In this paper a nonlinear, bistable, one d.o.f. system is considered (taken from [3, 2]), having the following equation of motion:

\[ \ddot{z} + 2\zeta \omega_n \dot{z} + f(z) = -F \cos(\sigma t + \varphi) \]  

(1)

where

\[ f(z) = (\omega_n^2 - c_1)z + c_3 z^3 = -\bar{c}_1z + c_3 z^3, \quad F = -\sigma^2 a \]  

(2)

and \( \bar{c}_1 = -(\omega_n^2 - c_1) \). An ad-hoc perturbation is applied to Eq. (1), in order to separate the contribution of the slow dynamics to the fast one. In particular, as a first step, a tri-linear approximation of \( f(z) \) is proposed (see Fig. 1a) and a quasi-stationary solution is analytically obtained, which is \( 2\pi \)-periodic and comprises jumps defined through the Heaviside step (see the phase plot in Fig. 1b). Then, the application of two steps of perturbation expansion to Eq. (1), when the quasi-stationary (slow) contribution is subtracted, allows one to get an approximation of the fast response. The recomposed solution, sum of the slow and fast contributions, is in good agreement with the numerical solution, as shown in Fig. 2.
Figure 1: Function $f(z)$ and tri-linear approximation (a); phase plot of the quasi-stationary solution (b).

Figure 2: Recomposed approximate solution (red line) and numerical solution (blue line).

References


Nonlinear Oscillatory Sonic Vacua

Alexander F. Vakakis¹, Leonid I. Manevitch²,

¹ Mechanical Science and Engineering  
University of Illinois  
1206 W. Green Street, Urbana, USA  
avakakis@illinois.edu

² Institute of Chemical Physics  
Russian Academy of Sciences  
Moscow 119991, Russia  
lmanev@chph.ras.ru

Abstract

We present a synopsis of analytical and numerical studies of a class of nonlinear dynamical systems designated as sonic vacua. The main feature of these systems is that their dynamics and acoustics do not possess any linear parts so the corresponding speed of sound (as defined in the context of classical acoustics) is zero. Accordingly, the nonlinear dynamics and acoustics of these media are essentially nonlinear (i.e., non-linearizable), and highly degenerate. This leads to interesting and highly complex nonlinear phenomena.

First, un-compressed ordered granular media are considered. These are ordered aggregates of individual particles (granules) interacting elastically in a highly nonlinear fashion. These highly discontinuous media have strongly nonlinear behavior owing to Hertzian interactions between granules when in compression, and separations of the same granules in the absence of it [1]. As a result the dynamics and acoustics of these media become non-smooth as granules regain contact through isolated or repetitive collisions. Moreover, due to the essential (non-linearizable) nonlinearities arising due to Hertzian interactions between granules, the acoustics of these media is highly adaptive to different applied excitations and static preloading, and in an acoustic analogy, they can be viewed as nonlinear ‘sonic vacua’ [1].

Figure 1. Propagating discrete breather in one of two coupled granular chains embedded in polyurethane matrix for excitation frequency at 500 Hz [2].

Despite their highly degenerate nonlinear dynamics and acoustics we show experimentally that harmonically excited ordered granular chains embedded in an elastic matrix possess acoustic pass and stop-bands, as well as propagating breathers [2]. Three different types of matrix – namely PDMS, polyurethane and geopolymer were considered, and both single and coupled granular chains were tested. In all cases the aforementioned nonlinear acoustic bands and breathers were robustly excited over varying frequency and amplitude ranges. Low-frequency acoustic pass-bands correspond to strongly nonlinear dynamics and pulse-like transmission in the granular media due to negligible effective compression; whereas high-frequency stop-bands are characterized by strongly localized standing waves in the media and almost linear dynamics.
due to strong effective compression, leading to complete elimination of transmitted wave components and acoustic filtering. Moreover, at intermediate frequency ranges breathers propagate, i.e., localized oscillating wavepackets separated by silent regions (cf. Figure 1); these results can be related to energy transfers between coupled granular chains.

Once the existence of propagating breathers in such ordered granular media is established, it is possible to design them for passive wave redirection and targeted energy transfer – TET [3]. In particular, considering two weakly coupled uncompressed granular chains mounted on linear elastic foundations, and designating one of the chains as the ‘excited’ chain and the other as the ‘absorbing’ one, we showed analytically and numerically that it is possible to redirect energy that is initially applied to the excited chain, to the absorbing one. This is achieved through passive TET caused by a macro-scale realization of the Landau-Zener tunneling quantum effect in space [3]. Numerical simulations fully validate the theoretical analysis, and the applications of these results to the design of a new type of nonlinear acoustic metamaterials with inherent energy redirection capacities are discussed.

The second type of nonlinear sonic vacua considered consists of chains of particles with next-neighbor interactions, undergoing in-plane nonlinear oscillations with fixed-fixed boundary conditions [4]. In the limit of low-energy predominantly transverse particle oscillations, we show that geometric nonlinearity generates nonlinear acoustic vacua, whereby the governing equations of motion possess strongly non-local nonlinear terms; these are generated by the near-uniform axial tension, which, in turn, is caused by the fixed boundary conditions. In fact, these strongly non-local terms can be regarded as time-dependent ‘effective speeds of sound’ for these media, which are completely tunable with energy. Another unexpected finding is that, the nonlinear normal modes (NNMs) of these systems are exactly identical to those of the corresponding linear chains, so they obey the same orthogonality conditions. A rich structure of resonance manifolds of varying dimensions can then be defined in the nonlinear dynamics of these systems, corresponding to resonance interactions between subsets of NNMs. The 1:1 resonance interactions were studied asymptotically and strong energy exchanges between modes were detected. Additional interesting resonance motions such as time-periodic oscillations with characteristics of both standing (close to the boundaries) and traveling waves (away from them) were detected in these highly degenerate dynamical systems.

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References

Detecting the Shilnikov scenario in a Hopf-Hopf bifurcation with 1 : 3 resonance

A. Steindl
Institute for Mechanics and Mechatronics
Vienna University of Technology
Getreidemarkt 9, A-1060 Vienna, Austria
Alois.Steindl@tuwien.ac.at

Abstract

We investigate the post-critical behaviour at a Hopf-Hopf interaction close to a 1 : 3 resonance. The third order normal form equations of the unfolded system are given by ((3))

\[
\begin{align*}
\dot{z}_1 &= (\lambda + i\omega)z_1 + A_1|z_1|^2z_1 + A_2|z_2|^2z_1 + A_3z_2^3, \\
\dot{z}_2 &= (\mu + 3i\omega + i\delta)z_2 + A_4|z_1|^2z_2 + A_5|z_2|^2z_2 + A_6z_1^3.
\end{align*}
\]

where \(\lambda, \mu,\) and \(\delta\) are the unfolding parameters and the complex valued coefficients \(A_j = c_j + id_j\) are obtained from the cubic expansion of the system at the bifurcation point. The terms with \(A_1, A_2, A_4\) and \(A_5\) would also show up at a non-resonant Hopf-Hopf interaction, whereas the last terms in both equations appear due to the 1 : 3-resonance.

Stationary solutions of the bifurcation equations

By introducing polar coordinates \(z_j = r_j \exp(i\varphi_j)\) the equations (1a) and (1b) could be reduced to two equations for the amplitudes \(r_j\) in the non-resonant case

\[
\begin{align*}
\dot{r}_1 &= (\lambda + c_1r_1^2 + c_2r_2^2)r_1, \\
\dot{r}_2 &= (\mu + c_4r_1^2 + c_5r_2^2)r_2.
\end{align*}
\]

The bifurcation behaviour of this system is well known, generically there exist 4 different stationary solutions:

- The trivial solution \(r_1 = r_2 = 0\), which is stable for \(\lambda < 0\) and \(\mu < 0\).

- The primary mode-1 solution \(r_1 \neq 0, r_2 = 0\); it exists in the parameter half plane \(\text{sign}(\lambda) = -\text{sign}(c_1)\) and is stable, if \(c_1 < 0\) and \(\mu + c_4r_1^2 < 0\).

- The primary mode-2 solution \(r_2 \neq 0, r_1 = 0\); it exists in the parameter half plane \(\text{sign}(\mu) = -\text{sign}(c_5)\) and is stable, if \(c_5 < 0\) and \(\lambda + c_2r_2^2 < 0\).

- The secondary mixed-mode solution \(r_1 \neq 0, r_2 \neq 0\). It exists in the sector

\[
\lambda + c_1\alpha + c_2\beta = 0, \\
\mu + c_4\alpha + c_5\beta = 0,
\]

with \(\alpha > 0\) and \(\beta > 0\) and bifurcates from the primary branches at pitchfork bifurcations.
• If $c_1c_5 - c_4c_2 > 0$ and $c_1c_5 < 0$, a tertiary quasiperiodic solution with 3 frequencies bifurcates from the mixed-mode solution.

In the resonant case the situation is quite different: Due to the terms $z_1^2z_2$ and $z_1^3$ in (1) the resonance angle $\psi = 3\varphi_1 - \varphi_2$ cannot be eliminated and the normal form dynamics lives in a 3-dimensional space.

Also the solution structure changes:

• Due to the presence of the term $z_1^3$ in (1b) the previous Mode-1 solution becomes a mixed mode solution, where the first mode acts similarly to an external excitation of the second mode. Close to the resonant region we observe a Duffing-like scenario.

• The pure Mode-2 solution still exists; at the secondary bifurcation a branch of mixed-mode solutions is born in an non-linear Mathieu-like scenario.

For a set of coefficients $A_j$, we followed the bifurcation branches numerically using the continuation software MatCont ([1]) and observed the occurrence of a Shilnikov scenario.

**Shilnikov bifurcation ([2])**

The Shilnikov bifurcation refers to the occurrence of a homoclinic solution at a saddle-focus: It can be shown, that this event may lead to an infinite family of horse-shoe maps, creating very complex dynamics. For example, there exists an infinite number of period-doubling sequences.

Since this bifurcation was observed numerically for a range of parameter values, we expect that it should be possible, to establish the existence of the homoclinic orbit at least approximately for the bifurcation equations. Especially an heteroclinic solution, which connects the primary solution branches close to the tertiary Hopf bifurcation in the non-resonant case, seems to be a good candidate for the homoclinic loop.

**References**


Resonant effects in stability of a rotor due to time-periodic terms

Alexei A. Mailybaev\textsuperscript{*}, Gottfried Spelsberg-Korspeter\textsuperscript{#}

* Instituto Nacional de Matemática Pura e Aplicada – IMPA, Rio de Janeiro, Brazil
alexei@impa.br

# Technische Universität Darmstadt, Germany
speko@dyn.tu-darmstadt.de

Abstract

Many effects in Rotordynamics can be explained with minimal phenomenological models that are designed to be easily solvable. The easiest model is probably the classical Laval rotor with only two degrees of freedom. In order to keep the equations simple usually symmetry of the bedding or the rotor is assumed, leading to equations with constant coefficients in a specific coordinate frame. However, it is intuitively clear that for an asymmetric rotor with asymmetric beddings no straightforward formulation of the equations of motion with constant coefficients is possible. This is also the case if an asymmetric rotor interacts with stationary contacts as, for example, in brakes or clutches.

These observations give the motivation to analyze the interaction of stationary (spatially fixed) and rotating (body fixed) asymmetries in a rotor system at the same time. This is especially interesting for the Laval rotor, since the static asymmetry can stabilize and rotating asymmetry destabilize the system in context of inner damping. Therefore, in this paper we consider an asymmetric Laval rotor in an asymmetric bedding with internal and external damping as well as frictional contact forces, see Fig. 1. Since the damping and contact forces are usually small compared to elastic restoring forces, tendencies and crucial effects can be simultaneously studied using analytical perturbation theory for eigenvalues. The methods are used in the context of Floquet theory for systems of equations which intrinsically feature time periodic coefficients.

We provide an analytical treatment for stability of a rotor with stationary and rotating asymmetries in the whole range of rotation speeds. In this theory, the resonant speeds represent the main mathematical novelty and difficulty. We show that due to intrinsic time-periodicity of system coefficients, the main resonance $\Omega \approx \omega$ between the rotation and natural frequency is associated to
Figure 2: Trefoil instability zones in parameter space for a rotor with no contact force and no dissipation. Bold lines show the first-order analytic approximation of the stability boundary and the gray regions represent the results of numerical simulations. The vertical axes describe the relative asymmetry in the stiffness (a) of the support and (b) of the rotor; $\Omega/\omega$ is the ratio of rotation and natural frequencies.

The semi-simple eigenvalue (Floquet multiplier) of multiplicity four. This represents an essentially higher degree of degeneracy as compared to the parametric resonance theory, where classical instability zones (tongues) result from double eigenvalues. We show that the perturbation equations can be resolved due to their specific structure associated with the axially symmetric unperturbed equations. At the same time, the quadruple eigenvalue leads to a highly nonlinear form of first-order stability conditions in terms of problem parameters, providing a new type of trefoil-shaped resonance zones instead of classical instability tongues, Fig. 2. We use the advantage of having a relatively simple system, which facilitates the comparison of analytical results with accurate numerical computation of stability regions.
Random Perturbations of Periodically Driven Nonlinear Oscillators

Nishanth Lingala*, N. Sri Namachchivaya*, Ilya Pavlyukevich#

* Department of Aerospace Engineering
University of Illinois
Urbana, Illinois 61801-2935, USA
lingala1, navam@illinois.edu

# Institut für Mathematik
Friedrich-Schiller-Universität Jena
Ernst-Abbe-Platz 2, 07743 Jena, Germany
ilya.pavlyukevich@uni-jena.de

Abstract

This paper develops a unified approach to study the dynamics of single degree of freedom systems excited by both periodic and random perturbations. The near resonant dynamics of such systems, in the presence of weak noise, is not well understood. We will study this problem in depth with the aim of discovering a common geometric structure in the phase space, and determining the effects of noisy perturbations on the passage of trajectories through the resonance zones. The main motivation for this work comes from the recent surge of research articles in harvesting of ambient broadband random vibration energy from diverse sources. This paper focuses on the “cantilever beam” type devices proposed in [5], which are used to convert small amplitude mechanical vibration from a specific application into an electrical energy source that could be used for electronic devices with low power requirements. Some of the recent work in energy harvesting have considered the possibility that stochastic resonance might help enable energy harvesting. Stochastic resonance is often defined as the optimal amplification of a weak periodic or narrow-band signal in a dynamical system by random noise. Hence, we also explore whether there is any relationship between the traditional stochastic resonance mechanism and energy harvesting. Some recent advances in stochastic dimensional reduction and associated graph valued processes [3] and stochastic resonance [2] not only yield additional motivation for this class of models but also bring techniques that lead to the understanding of higher dimensional (n > 3) realistic systems.

Prototypical beam type nonlinear energy harvesting models contain double well potentials, external and/or parametric periodic forcing terms, damping and ambient broadband additive noise terms (for example, mean zero, stationary, independent Gaussian white noise processes). Hence the general form of the equations studied here is given by

\[ \ddot{q}_t + \frac{\partial U}{\partial q}(q_t) + G(q_t, \dot{q}_t) \dot{q}_t = \eta \cos(\nu t) q_t + \alpha \cos(\nu t) + \sigma \xi(t), \tag{1} \]

where \( q \in \mathbb{R} \) represents a generalized coordinate and \( U: \mathbb{R} \to \mathbb{R} \) is a double well potential and accounts for the conservative force and \( G \) represents the dissipative terms. More precisely, we define \( U(q) \overset{\text{def}}{=} -\frac{1}{2} \mu q^2 + \frac{1}{4} q^4, \mu > 0 \) and \( G(q) \overset{\text{def}}{=} (\delta - \beta q^2), \delta > 0, \beta \geq 0 \). For the weakly nonlinear (\( \epsilon \) order cubic nonlinearity) noisy Duffing–van der Pol–Mathieu equation (1), Namachchivaya and Sowers [4] developed a unified approach by a clever treatment in a neighborhood of the separatrix where the unperturbed orbits have arbitrarily long periods. However, such an analysis will not be fruitful in explaining energy harvesting, since for optimal harvesting, the dynamics need to have large rotations, not just oscillations confined to one of the potential wells. In this paper, we extend the work of [4] to include more general (strongly) nonlinear systems and obtain analytical results for a low-dimensional model of (1) following the approaches and ideas given in [3] and [2].

In the deterministic case, the system’s frequency \( \omega(I) \) depends on the amplitude/action \( I \), and the resonance conditions is \( \kappa_1 \omega(I) + \kappa_2 v = 0, (\kappa_1, \kappa_2) \in \mathbb{Z}^2 - (0,0) \). Close to resonance, the trajectory moves slowly while preserving the resonance condition and may leave the resonance surface.
close to $I_{K_1:K_2}$ after a large time interval. It has been shown both numerically and experimentally under certain condition, the stochastic trajectories of (1) with $\beta = 0$ reach their largest amplitude with noise assisted jumps between the domains of attractions as depicted in Fig. 1. However, a close scrutiny shows the noise induced transitions in the limit of small noise intensity, is very complicated. Unraveling these transitions between stable limit cycles (quasi-periodic) and the study of exit times from the domains of attractions are the focus of this theoretical study.

Figure 1 shows the most probable path of the noise induced transitions in a noisy, damped, Mathieu–Duffing–Oscillator. The probability of these transitions can be estimated in terms of the so-called quasi-potentials defined by means of action functionals (for details see [1]). Roughly, these quasi-potentials measure the work done by the noise in order to travel between different points in the domains of attractions of the stable limit cycles. Only the work done along the noisy trajectory (lies within the thin tube depicted in Fig. 1) on the way “out” of the center domain of attraction contributes to the quasipotential; the way “towards” the domain of attraction of the stable limit cycle requires no work.

The purpose of this paper is three fold:

i to develop stochastic techniques, based on the martingale problem, large deviation theory and stochastic resonance to understand the dynamics of stochastically perturbed two-dimensional weakly Hamiltonian systems close to resonances;

ii to explain the energy harvesting mechanism and determine analytically the frequency $\nu$ for optimal harvesting (provide a mechanism by which the harvesting by weak input signal can meaningfully be enhanced);

iii to clarify the relevance of stochastic resonance in the context of energy harvesting by exploring the relationship between these two mechanisms.

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**References**


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Strongly Nonlinear Oscillators with a Zero, Negative and Positive Linear Stiffness Term: Generalised Perturbation Techniques for Free and Forced Systems

Ivana Kovacic

Faculty of Technical Sciences
University of Novi Sad
Novi Sad, Serbia
ivanakov@uns.ac.rs

Abstract

This is a two-part study concerned with the quantitative techniques for obtaining the response of nonlinear oscillators whose restoring force $F_r$ is modelled by the following general expression:

$$F_r(x) = k x + \varepsilon K \text{sgn}(x)|x|^\alpha,$$

where $x$ is a non-dimensional displacement; $k$ and $K$ are constants; $\varepsilon$ is assumed to be a positive constant, but not necessarily small; $\alpha$ is an arbitrary positive real number. This model includes four types of restoring force: i) purely nonlinear ($k = 0$, $K > 0$); ii) with a negative linear term and a positive power-form nonlinear term ($k = -1$, $K > 0$), yielding a bistable oscillator; iii) with a positive linear term and a positive power-form nonlinear term ($k = 1$, $K > 0$), which corresponds to a hardening restoring force, and iv) with a positive linear term and a negative power-form nonlinear term ($k = 1$, $K < 0$), which corresponds to a softening restoring force.

In the first part, free conservative oscillators with the restoring force (1) are considered:

$$\ddot{x} + k x + \varepsilon K \text{sgn}(x)|x|^\alpha = 0,$$

where the overdots denote differentiation with respect to $t$. Following the qualitative discussion related to the corresponding potential well and the values of the overall initial energy, the conditions for the periodic orbits around the origin are defined. To find the corresponding solution for motion for all four types of nonlinear oscillators of interest, the following new parameter is introduced [1]:

$$p = \frac{\varepsilon K b_{1a} a^{a-1} - \varepsilon K c^2 a^{a-1}}{k + \varepsilon K b_{1a} a^{a-1}} = \frac{\Delta \varepsilon K a^{a-1}}{k + \varepsilon K b_{1a} a^{a-1}},$$

where $a$ is the amplitude of vibration, $\Delta = b_{1a} - c^2$, $b_{1a} = 2 \Gamma(1 + \frac{2}{a+1}) \Gamma(\frac{1}{a+1})$ and $c = \left[\frac{\Gamma(\frac{a+1}{2})}{\Gamma(\frac{a+1}{2})} \right]^{1/2}$. Note that both $b_{1a}$ and $c$ depend on the power of nonlinearity $\alpha$. They are related, respectively, to the frequency of vibration obtained from the first order balancing and the period calculated from the energy conservation law (more details are given in [1]). It is also important to note that the parameter $p$ stays small regardless of the value of $\varepsilon K a^{a-1}$: when $\varepsilon K a^{a-1} \rightarrow 0$, then $p \rightarrow p^* = \Delta / b_{1a}$. For $\alpha \in (0,1]$, one can calculate $p^* \in (0.0311,0]$, while for $\alpha \in [1,5]$, one has $p^* \in (0,0.1076]$. Thus, given these values, the parameter $p$ can be used as a new small perturbation parameter.

Now, new time is introduced as $\tau = \omega t$, with $\omega$ being the square of the unknown frequency expanded in power series in $p$ as follows:

$$\omega^2 = \left[k + \varepsilon K b_{1a} a^{a-1}\right] \left[1 + p\omega_1 + p^2\omega_2\right],$$

where $\omega_1$ and $\omega_2$ will be selected in the subsequent procedure to eliminate secular terms. By
expressing $\epsilon$ from Eq. (3) and using Eq. (4), one can transform Eq. (2) into:

$$x^* + x = p \left( -\omega_1 x^* + x \frac{b_{1}\alpha}{\Delta} - \frac{1}{a^{2} - 1} \frac{\Delta}{\Delta} \cdot \text{sgn}(x) \cdot \frac{x^\alpha}{x^\alpha} \right) - p^2 \omega_2 x^*, \quad (5)$$

where the primes denote differentiation with respect to $\tau$. Equation (5) corresponds now to a perturbed harmonic oscillator and its left-hand side is fixed regardless of the class of the original oscillator considered. A thorough investigation of the validity and accuracy of the solutions obtained by applying the Lindstedt-Poincaré in the second approximation to Eq. (5) is carried out, and one of them is shown in Figure 1a.

The second part is concerned with forced non-conservative oscillators governed by [2]:

$$\ddot{x} + \epsilon \left( \zeta_1 + \zeta_2 x^2 + \zeta_3 x^4 \right) \dot{x} \cdot \text{sgn}(x) + F_r(x) = \epsilon f \cos \Omega t, \quad (6)$$

where $\zeta_1, \zeta_2$, and $\zeta_3$ are positive constants; $\beta$ is an arbitrary positive real number. The model of the non-conservative force considered covers linear, quadratic and non-polynomial damping as well as displacement-dependent damping. A detuning parameter $\sigma$ is introduced as a measure of a frequency deviation from the backbone curve, as follows:

$$\Omega^2 = \left( k + \epsilon K b_{1}\alpha a^{-1} \right) (1 + p\sigma). \quad (7)$$

Equation (6) is transformed into

$$x^* + x = p \left( -\alpha x^* + \left( \zeta_1^* + \zeta_2^* x^2 + \zeta_3^* (x^3)^2 \right) \dot{x} \cdot \text{sgn}(x) + x \frac{b_{1}\alpha}{\Delta} - \frac{1}{a^{2} - 1} \cdot \text{sgn}(x) \cdot \frac{x^\alpha}{x^\alpha} + f^* \cos \tau \right), \quad (8)$$

where

$$\zeta_1^* = \frac{\zeta_1 \Omega^\beta}{K a^{\alpha - 1} \Delta}, \quad \zeta_2^* = \frac{\zeta_2 \Omega^\beta}{K a^{\alpha - 1} \Delta}, \quad \zeta_3^* = \frac{\zeta_3 \Omega^\beta + 2}{K a^{\alpha - 1} \Delta}, \quad f^* = \frac{f}{K a^{\alpha - 1} \Delta}. \quad (9)$$

The first-order differential equations for the amplitude and phase are derived by the method of multiple scales. Steady-state solutions and the corresponding amplitude-frequency relationship are also obtained, plotted and examined for a variety of cases, one of which is shown in Figure 1b.

**Figure 1.** Comparisons of the approximate solution obtained (solid line) with the numerical solution (black dots) for $\alpha=2/3, \epsilon=1, k=0, K=1$: a) free vibration with $a=1$; b) forced vibrations $\zeta_1 = 0.1, \zeta_2 = \zeta_3 = 0, \beta = 1, f=1$.

**References**


Highly nonlinear liquid surface waves in the dynamics of the fluid balancer

Mikael A. Langthjem∗, Tomomichi Nakamura#

∗ Faculty of Engineering
Yamagata University
Jonan 4-3-16, Yonezawa,
992-8510 Japan
mikael@yz.yamagata-u.ac.jp

# Department of Mechanical Engineering
Osaka Sangyo University
Nakagaito 3-1-1, Daito-shi, Osaka,
574-8530 Japan
t-nak@mech.osaka-sandai.ac.jp

Abstract

The present paper is concerned with the dynamics of a so-called fluid balancer, as used in rotating machinery (including contemporary household washing machines) to eliminate, or limit, the undesirable effects of unbalance mass. This is a hula hoop ring-like structure, containing a small amount of liquid which, during rotation, is spun out to form a thin liquid layer on the inner surface of the ring. This liquid layer is, at supercritical rotor angular speeds \( \Omega > \Omega_{cr} \), able to counteract unbalance mass in an elastically mounted rotor, as sketched in Figure 1.

In a recent paper we were able to give an analytical explanation of the basic mechanism of the balancer. In words, the balancer works as follows. For a rotor with an unbalance mass \( m \), and without fluid, it is well known that the unbalance mass is in the direction of the rotor deflection at sub-critical rotation speeds, \( \Omega < \Omega_{cr} \), and opposite the direction of the rotor deflection at super-critical rotation speeds, \( \Omega > \Omega_{cr} \) (when seen from a rotating coordinate system, attached to the rotor). A perturbation analysis of the problem involving fluid shows that the mass center of the fluid layer \( M \) is in the direction of the rotor deflection for any rotation speed. In this way a surface wave on the fluid layer can counterbalance an unbalanced mass.

The present work presents a careful analysis of highly nonlinear fluid layer distributions (which were not considered in [1]) and their bearing on the fluid balancer dynamics.

In terms of non-dimensional variables, the fluid layer thickness perturbation \( \kappa_0 \) (made non-dimensional with the average fluid layer thickness \( h_0 \)) is described by the equation

\[
\mathcal{A} \frac{\partial \kappa_0}{\partial \xi} + \mathcal{B} \kappa_0 \frac{\partial \kappa_0}{\partial \xi} = \mathcal{C} \frac{\partial^3 \kappa_0}{\partial \xi^3} - \mathcal{D} \frac{\partial^2 \kappa_0}{\partial \xi^2} + \mathcal{E} \kappa_0^2 = \chi \sin \xi.
\]

(1)

The parameter \( \xi \) is a non-dimensional ‘traveling wave’ variable, defined by \( \xi = \theta - (\omega - \Omega) t \). Here \( \theta \) is the angular coordinate in a polar coordinate system \( (r, \theta) \) attached to the rotor, \( \omega \) is the

angular whirling speed of the fluid, and $\Omega$ is again the angular speed of the rotor. $\omega$ is assumed to be slightly different from $\Omega$, that is, the fluid is assumed to be in a state of asynchronous whirl. Finally, $t$ is the time. On the right hand side, $x_*$ is the rotor deflection in the radial direction $r$, in the just mentioned rotor-attached coordinate system $(r, \theta)$.

Returning to the left hand side of (1), $A_1$ represents the wave speed (relative to the $\xi$ ‘axis’), $B_1$ represents a non-linear wave speed correction, $C_1$ represents dispersion, $D_1$ represents viscosity (internal dissipation), and $E_1$ represents boundary friction/dissipation.

Equation (1) is a forced Korteweg-de Vries-Burgers equation. Without dissipation ($D_1 = E_1 = 0$) and external forcing ($x_* = 0$) it reduces to the Korteweg-de Vries equation. The Burgers equation is obtained with $C_1 = E_1 = 0$ and $x_* = 0$.

Reference [1] considers multiple scales approximations of so-called cnoidal wave (soliton-like) solutions, as sketched in Figure 2(a). The present paper is concerned mainly with the hydraulic jump solution sketched in Figure 2(b). This type of solution is found when the parameter $C_1$ in (1) is small, and $B_1$ relatively large. We are then faced with a singular perturbation problem which remains nonlinear as the small parameter $\varepsilon = C_1 \to 0$. A simplified version of the problem was considered in [2], assuming simply that $C_1 = D_1 = E_1 = 0$. In the present work we obtain, and compare, solutions to the ‘full’ problem by (i) the method of matched asymptotic expansions and (ii) the method of multiple scales, in the special form known as the method of Kuzmak-Luke [3].

![Figure 2](image_url)

Figure 2: Sketch of possible appearances of the fluid layer within the rotating fluid balancer. (a) Cnoidal wave solution. (b) Hydraulic jump solution.

The main point of the present work is, again, to investigate how the hydraulic jump solution of Figure 2(b) has bearing on the fluid balancer dynamics. Equation (1) is solved assuming the rotor deflection $x_*$ to be known. Based on the knowledge of the fluid layer perturbation function $k_0(\xi)$ the fluid pressure distribution is evaluated; then the fluid force acting on the rotor. Next, the rotor equation of motion (with motion-dependent fluid force terms) is solved. Since the fluid is in a state of asynchronous whirl, this equation will take the form of a forced Mathieu equation. (A deflection-independent forcing term is due to the unbalance mass.) A perturbation expansion-type solution is obtained by employing yet again the method of multiple scales.

References

Analysis of Engineering Systems by Normal Form Theory

D. Hochlenert

Faculty of Mechanical Engineering and Transport Systems
Technische Universität Berlin
Einsteinufer 5, 10587 Berlin, Germany
daniel.hochlenert@tu-berlin.de

Abstract

There exists a permanent trend of widening the operating range of engineering systems in order to optimize the operating state. It is even accepted to run systems under unstable operating conditions to achieve this objective. In the context of turbochargers for instance, small self-excited vibrations are tolerated to reach higher speeds of rotation. Furthermore, self-excited vibrations in general play an important role for the question of realizable operating states. Related engineering examples are brake squeal, the directional stability of railway vehicles, wear and instability issues of paper calenders, and chatter of (micro) milling processes. Therefore, a detailed stability analysis in addition to the identification of characteristic frequencies and modes of vibration are fundamental steps for the investigation of the dynamics of engineering systems.

In the context of dynamical systems the common stability definition is LYAPUNOV stability. However, LYAPUNOV’s stability definition only deals with infinitesimal disturbances of initial conditions of the considered solution and therefore might yield practically insufficient stability statements. In order to be of practical relevance, i.e. to be a realizable operating state for example, the corresponding LYAPUNOV-stable solution needs to have a sufficiently large domain of attraction [1, 3]. This discrepancy between LYAPUNOV stability and stability from a practical point of view is well known. The actual problem is, that the high dimensional systems of differential equations resulting from a detailed mathematical mechanical model are usually investigated by LYAPUNOV’s indirect method, i.e. by an eigenvalue analysis of the linearized system. This is insufficient, if the original nonlinear dynamical system features neighboring stable and unstable solutions and/or bifurcations with respect to the variation of parameters. Moreover, the analysis of the linearized system provides no information on the domains of attraction of the solutions.

The present analysis is devoted to a systematic dimension reduction of nonlinear dynamical systems allowing for a semi-analytical stability and bifurcations analysis. According to STERNBERG’s linearization theorem [2] and the resonance condition

\[ \lambda_j = m_1 \lambda_1 + m_2 \lambda_2 + \ldots + m_n \lambda_n, \quad j = 1, \ldots, n, \quad m_i \geq 2, \text{ integer} \]  

for the system’s eigenvalues \( \lambda_i \), it is shown that a straightforward coordinate transformation implicitly decouples the nonlinear equations of motion and yields the bifurcation equation in normal form. The normal form transformation is then used to approximate the domain of attraction directly and to construct adapted LYAPUNOV functions which can be used for calculating a subdomain of the exact domain of attraction.

As an engineering example the directional stability of a railway wheelset is investigated. Special regard is given to the contact between wheel and rail. ORE S1002 profiles of the wheels and UIC 60 profiles of the rails are considered. The contact forced are modeled according to the linear theory of KALKER. The nonlinearities are expressed as polynomials of the generalized coordinates.
up to fifth degree. The equations of motion are analyzed by normal form theory regarding the stability and domains of attraction of the trivial solution, i.e. the directional stability. The results obtained by normal form theory are compared to numerical investigations.

References


Nonlinear Dynamics Orbital Instabilities and Transient Chaos in Magnetic Resonance Force Microscopy

E. Hacker, S. Pandey, O. Gottlieb

Department of Mechanical Engineering
Technion – Israel Institute of Technology
Haifa 3200003, Israel
oded@technion.ac.il

Abstract

Magnetic resonance force microscopy (MRFM) is an imaging technique that enables acquisition of three-dimensional magnetic images at nanometer scales, and has been adapted for the detection of the magnetic spin of a single electron [5]. It is based on combining the technologies of magnetic resonance imaging (MRI) with atomic force microscopy (AFM). In conventional MRI devices the electronic spins are detected by measuring their magnetic induction using an inductive coil as an antenna. However, in MRFM the detection is implemented mechanically using a microcantilever to directly detect a modulated spin gradient force between the sample spins and a ferromagnetic particle attached to the tip of the cantilever. Furthermore, MRFM techniques have recently been used to estimate dynamic nuclear polarization in addition to detection of electron spin magnetization [2]. While MRFM systems are receiving a growing amount of interest, to date, a comprehensive theoretical treatment is still lacking. Existing models are based on simplistic lumped-mass reductions that include linear estimates of cantilever stiffness and damping complemented by a nonlinear approximation of the magnetic force [1].

We thus consistently formulate the MRFM nonlinear initial-boundary-value problem (IBVP) combining the microcantilever dynamics and the time-dependent magnetic moments of the spin using the Bloch equations for magnetization (Fig.1 left). We reduce the IBVP to a finite-order nonlinear dynamical system using a Galerkin modal projection [3] and investigate stability of the equilibrium solution of the respective autonomous system for different values of a magnetization parameter to reveal both pitchfork and saddle-node bifurcations (Fig.1 right). We employ multiple scales asymptotics to deduce the frequency response of a limiting adiabatic configuration (Fig.2 left) and validate the hysteretic form of the response in a macroscale experiment with a permanent tip magnet that is attracted to a magnet located on the sampled surface (Fig.2 right). We determine existence of internal resonances in a three-mode configuration [4] and of lengthy chaotic-like transients for low damping near primary resonance (Fig.3 left). The spectra of the magnetic transients (Fig.3 right) were found to be dense, whereas the spectra of the displacement included the excitation frequency and a m/n=5/4 ultra-subharmonic reminiscent of a secondary bifurcation due to hard excitation.

Figure 1. Definition sketch of the MRFM setup (left). Cantilever equilibrium vs. a normalized magnetization parameter depicting pitchfork and saddle-node bifurcation (right).
Figure 2. Asymptotic MRFM frequency response from the adiabatic reduced-order model (left). Experimental frequency response from a macroscale magnetoelastic dipole interaction (right).

Figure 3. Time series of cantilever displacement and magnetic moment components of a chaotic transient solution (left). Power spectra of the chaotic transient solution (right).

References


Reduced Order Models for the Nonlinear Dynamic Analysis of Shells

Paulo B. Gonçalves*, Frederico M. A. da Silva#, Zenón Del Prado#

*Civil Engineering Department
Catholic University, PUC-Rio
22453-900 Rio de Janeiro, RJ, Brazil
paulo@puc-rio.br

#School of Civil Engineering
Federal University of Goiás, UFG
74605-200, Goiânia, Goiás, Brazil
[silvafma, zenon]@ufg.br

Abstract

The non-linear dynamic analysis of continuous systems, such as thin plates and shells, is a problem of relevance in many engineering fields. The finite element method is the most used approach for nonlinear dynamic analyses of these structures. However, the computational effort is very high. As an alternative to complex numerical approaches, analytical methods using simplified models can be successfully used to understand the main nonlinear features of the problem and may constitute efficient tools in the initial design stages. For plates and shells, the derivation of efficient reduced order models is in fact essential due to the complex nonlinear response of these structures. The usual procedure is to reduce the partial differential equations of motion of the continuous system to an approximate system of time-dependent ordinary differential equations of motion, which are in turn solved by numerical methods or, approximately, by perturbation procedures. However, the use of inappropriate modal expansions usually leads to misleading results or may require a rather large number of terms. The aim of the present work is to show how the application of perturbation analyses can be used to derive precise low order models for plates and shells, by capturing the influence of modal couplings and interactions. The Donnell nonlinear kinematic relations for thin shells are [1]:

\[
\begin{align*}
\varepsilon_x &= \frac{u_y}{A} + \frac{A_y}{AB} + \frac{w}{R_x} + \frac{1}{2} \beta_x^2; \\
\varepsilon_y &= \frac{v_x}{B} + \frac{B_x}{AB} + \frac{w}{R_y} + \frac{1}{2} \beta_y^2; \\
\gamma_{yx} &= \frac{uv}{A} + \frac{B_x v + A_y u}{AB} + \beta_x \beta_y; \\
\chi_x &= \frac{\beta_{x,y}}{A} + \frac{A_y \beta_x}{AB}; \\
\chi_y &= \frac{\beta_{y,x}}{B} + \frac{B_x \beta_y}{AB}; \\
\gamma_{xy} &= \frac{\beta_{x,y}}{A} + \frac{A_x \beta_y}{AB} + \frac{B_y \beta_x}{AB} + \frac{A_y \beta_x}{AB} + \frac{B_x \beta_y}{AB}; \\
\beta_x &= \frac{w_y}{A} + \frac{u_x}{R_x}; \\
\beta_y &= \frac{w_x}{B} + \frac{v_y}{R_y}
\end{align*}
\]

(1)

where \( \varepsilon_x, \varepsilon_y, \) and \( \gamma_{xy} \) are the extensional and shearing strain components at a point on the shell middle surface, \( \beta_x, \beta_y \), are the rotations, \( \chi_x, \chi_y, \) and \( \chi_{xy} \), are, respectively, the curvature changes and twist, \( u \) and \( v \) are the in-plane displacements in the \( x \) and \( y \) directions respectively, \( w \) is the transversal displacements, \( A \) and \( B \) are the Lamé coefficients, \( R_x \) and \( R_y \) are the principal radii of curvature, which define the shell geometry.

Based on (1), the nonlinear equations of motion for the undamped, unforced thin shell can be written in terms of its displacement vector \( \mathbf{U} = \{ u, v, w \}^T \) as:

\[
\mathbf{L}(\mathbf{U}) - \mathbf{U}_{xx} = \delta \mathbf{D}_1(\mathbf{U}) + \delta^3 \mathbf{D}_2(\mathbf{U})
\]

(2)

where \( \mathbf{L}(\mathbf{U}) \) is the matrix of linear differential operators, \( \delta \) is an appropriate small perturbation parameter, \( \mathbf{D}_1(\mathbf{U}) \) is a vector of quadratic terms and \( \mathbf{D}_2(\mathbf{U}) \) is a vector of cubic terms. Other common shell theories, such as Sanders and Koiter theory, lead to similar results. One assumes...
that the components of the displacement vector $\mathbf{U}$ can be expanded in terms of the perturbation parameter $\delta$ as:

$$
\begin{align*}
    u &= \sum_{i=0}^{\infty} \delta^i U_i(x, y) \\
    v &= \sum_{i=0}^{\infty} \delta^i V_i(x, y) \\
    w &= \sum_{i=0}^{\infty} \delta^i W_i(x, y)
\end{align*}
$$

(3)

Substituting Eq. (3) into Eq. (2), collecting terms of the same order in $\delta$ and equating them to zero, one obtains a set of partial differential equations. The solution of the first set of equations with the appropriate boundary conditions is simply the linear vibration modes, $\mathbf{U}^0$. Substitution $\mathbf{U}^0$ into the second equation set, leads to a system of non-homogeneous differential equations, which is linear in $\mathbf{U}^1$. The first order solution $\mathbf{U}^1$ arises from the quadratic nonlinearity and is the main responsible for the in-out asymmetry of a shell nonlinear displacement field. Substituting $\mathbf{U}^0$ and $\mathbf{U}^1$ in the third equation set, one can obtain the second order modes $\mathbf{U}^2$, which captures the influence of the cubic nonlinearity. These equations have a well-defined pattern, in a way that one could continue developing higher order modes up to the desired order. Finally, by inspecting the solution for $\mathbf{U}^1$, $\mathbf{U}^2$, $\mathbf{U}^3$, ..., $\mathbf{U}^N$, a general solution for the displacement field can be derived. Shells exhibit a dense frequency spectrum, especially in the lowest frequency range. In addition, for shells of revolution, due to the circumferential symmetry, frequencies occur in pairs. So, multiple internal resonances may occur. To obtain a consistent modal solution for these problems, the analysis must consider as the modal initial solution $\mathbf{U}^0$ the sum of all interacting modes (seed modes) in the perturbation procedure. The results for various structural systems have shown that, using the present procedure, a precise model can be obtained using a small number of degrees of freedom thus enabling the use of a large number of numerical and approximate analytical tools developed for nonlinear dynamical systems [2, 3]. The solution of problems involving modal coupling, as illustrated in Figure 1, circumferential symmetry, multimode interaction and various $m$-to-$n$ internal resonances are investigated in this work considering shells of revolution and panels.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure.png}
\caption{Influence of first (02, 20, 22) and second order modes (31, 13, 33) obtained by the perturbation procedure on the frequency-amplitude relation of a cylindrical shell. (a) convergence, (b) coupling effect: seed mode (11) plus selected first and second order modes.}
\end{figure}

References


On the dynamic balancing of a planetary moving rotor using a passive pendulum type device

Olga Drozdetskaya\textsuperscript{\ast}, Alexander Fidlin\textsuperscript{\ast}

\textsuperscript{\ast}Institute for Engineering Mechanics  
Karlsruhe University of Technology  
Kaiserstraße 10, D- 76131 Karlsruhe, Germany  
olga.drozdetskaya@kit.edu, alexander.fidlin@kit.edu

Abstract

The phenomenon of self-balancing of rigid rotors is well known and investigated for rotors with fixed bearings \cite{1}, \cite{3}. However, in some technical devices the rotor performs complex motions. An example of such system is a computed tomography scanner. Its anode rotates very fast in the housing of the X-ray tube. At the same time the X-ray tube itself rotates rather slowly around the patient’s body. It is very important for CT scanner to keep the minimal possible level of vibrations in order to obtain good image quality. The objective of this paper is to investigate how and to which extent the self-balancing devices can be used for reducing vibrations in a planetary moving rotor.

Consider the system in Figure 1 (a) representing a rotating rotor on a rigid carrier.

![Diagram](image)

Figure 1: Unbalanced rotor on the rigid carrier (a) and the stationary configurations of the rotor with pendulums (b)-(e).

The rotor of mass $M$ is fixed on the end of the carrier which length is $R$. The other end of the carrier is elastically suspended with radial spring-dampers of a certain stiffness $c$ and damping $\beta$. The carrier rotates around it’s point of suspension with a constant velocity $\Omega$. At the same time
the rotor rotates around its symmetry axis with a given velocity $\omega$. Its centre of mass has an offset $\gamma$ relative to the rotation axis. Two pendulum balancers of mass $m$, moment of inertia $J$ and length $r$ are placed on the rotation axis of the rotor.

The equations of motion of the whole system (1) - (3) can be split into two groups. The first two equations describe the radial vibrations of the carrier system. The last two equations describe the phases of the pendulums. Herewith it has been assumed that the damping both in the oscillation subsystem $\beta$ and in the rotation degrees of freedom for pendulums $\beta\phi$ is not small.

\[(M + 2m)\ddot{x} + \beta \dot{x} + x = (M + 2m)R\Omega^2 \cos \Omega t + M\gamma \omega^2 \cos \omega t + mr \sum_{i=1}^{2} (\dot{\phi}_i \cos \phi_i + \dot{\phi}_i \sin \phi_i)\]  
\[(M + 2m)\ddot{y} + \beta \dot{y} + y = (M + 2m)R\Omega^2 \sin \Omega t + M\gamma \omega^2 \sin \omega t + mr \sum_{i=1}^{2} (\dot{\phi}_i \sin \phi_i - \dot{\phi}_i \cos \phi_i)\]  
\[(J + mr^2)\ddot{\phi}_i + \beta\phi_i (\phi - \omega) = mrR\Omega^2 (\omega t - \phi_i) + mr(x \dot{x} \sin \phi_i - \dot{y} \cos \phi_i); i = 1, 2\]  

The approximate asymptotic solution has been obtained using averaging technique for the strongly damped systems [2]. Four stationary solutions have been obtained for the system. The possible stationary geometrical configurations of the rotor with the self-balancing device are shown in Figure 1 (b)-(e). Only the first configuration provides the compensation of unbalance. The stability investigations of the obtained solutions have shown that the configuration (b) is the only stable one in the overcritical domain of rotation speeds of rotor. The solution (c) is the only stable one in the undercritical domain. The solutions (d) and (e) are unstable. The improved first-order approximation has been obtained for the compensating configuration of the system:

\[\phi_1 = \omega t + \alpha + \frac{R\Omega^2}{r\omega^2 \sqrt{1 + b^2 (1 + \frac{J}{mr^2})}} \cos((\Omega - \omega)t - \alpha + \arctan \frac{1}{b});\]  
\[\phi_2 = \omega t - \alpha + \frac{R\Omega^2}{r\omega^2 \sqrt{1 + b^2 (1 + \frac{J}{mr^2})}} \cos((\Omega - \omega)t + \alpha + \arctan \frac{1}{b});\]  
\[\alpha = \arctan \left( -\frac{4m^2 r^2}{M^2 - \gamma^2} - 1 \right); \quad b = \frac{\beta\phi}{\omega(mr^2 + J)}\]  

Hence the self-balancing of pendulum type is effective for a planetary rotor in the overcritical speed domain. However the balancing is limited by the centrifugal forces causing small vibrations of pendulums. The expressions for the residual amplitudes of vibrations are obtained in dependency on the velocity of the planetary motion. Analytic results match very well with numeric simulations when the velocity of planetary motion is sufficiently small.

References


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The Method of Varying Amplitudes for Solving (Non)linear Problems Involving Strong Parametric Excitation

Vladislav S. Sorokin*,#, Jon J. Thomsen*

* Department of Mechanical Engineering
Technical University of Denmark
Nils Koppels Allé, Building 404, 2800
Kgs. Lyngby, Denmark
vladsor@mek.dtu.dk

# Institute of Problems in Mechanical Engineering RAS
V.O., Bolshoj pr. 61, St.Petersburg,
199178, Russia
slavos87@mail.ru

Abstract

Parametrically excited systems appear in many fields of science and technology, intrinsically or imposed purposefully; e.g. spatially periodic structures represent an important class of such systems [4]. When the parametric excitation can be considered weak, classical asymptotic methods like the method of averaging [2] or multiple scales [6] can be applied. However, with many practically important applications its simplification is inadequate, e.g. with spatially periodic structures it restricts the possibility to affect their effective dynamic properties by a structural parameter modulation of considerable magnitude. Approximate methods based on Floquet theory [4] for analyzing problems involving parametric excitation, e.g. the classical Hill’s method of infinite determinants [3,4], can be employed also in cases of strong excitation; however, with Floquet theory being applicable only for linear systems, this is impossible or rather cumbersome for combined parametric and direct excitation, or with nonlinearity.

The present work employs a novel approach, the Method of Varying Amplitudes (MVA) [7], for solving linear and nonlinear problems involving combined direct excitation and strong parametric excitation. This approach is inspired by the method of direct separation of motions [1]; it is strongly related to Hill’s method of infinite determinants [3,4] and the method of space-harmonics [5], and may be considered a continuation of the classical methods of harmonic balance [3] and averaging [2]. It implies a harmonic series solution with varying amplitudes, but in contrast to averaging methods, the amplitudes are not required to vary slowly. Thus the MVA does not assume the presence of a small parameter in the governing equations, or any restrictions on the sought solution. To illustrate the method several problems are considered:

The first problem is vibration suppression in predefined regions of a string subjected to distributed time-periodic loading, by continuous spatial cross-section modulation; this could be relevant for, e.g., oscillations of transmission lines, suspension bridges, and stay cables under rain and wind. Employing the effect of parametric attenuation [3], the problem is reduced to a forced linear Mathieu equation. As a result optimal parameters for the string cross-sectional area modulation are determined for the cases of harmonically, uniformly, and arbitrarily distributed load.

The second problem is the determination of eigenproperties for a Bernoulli-Euler beam with periodically and continuously varying spatial properties, as is relevant e.g. for risers and rotor blades. The corresponding governing equation is, in non-dimensional form:

\[
(1 + \chi_I \sin x)\phi^{\prime\prime} - \delta(1 + \chi_A \sin x)\phi = 0,
\]

where $\phi=\phi(x)$ represents beam deflection, $\chi_I$ and $\chi_A$ modulation amplitudes for the beam stiffness and mass per unit length respectively, and $\delta$ the squared non-dimensional frequency. As a
result the dispersion relation and eigenproperties are determined, and it is shown that such non-
uniform structures are able to sustain long-wave oscillations at comparatively high frequencies.

Thirdly, the effect of weak nonlinearity on the dispersion relation and the frequency band-gaps of a periodic Bernoulli-Euler beam is examined; this is relevant, since applications may demand
effects of nonlinearity on structural response to be accounted for. The corresponding governing non-dimensional equation takes the following form:

$$(1 + \chi_1 \sin x) \left( \frac{\varphi'' + \mu (\varphi'')^2 \varphi'}{\varphi'} \right) - \delta (1 + \chi_2 \sin x) \varphi = 0,$$

(2)

where the parameter $\mu$ defines the nonlinearity of the beam stress-strain relation, and the over-
bar denotes complex conjugation. As a result a shift of band-gaps to a higher frequency range is
revealed, while the width of the band-gaps appears relatively insensitive to (weak) nonlinearity.

Fourthly we analyze the response of perfectly tuned or slightly detuned nonlinear parametric
amplifiers; this is of interest e.g. for micro and nanosystems. The model equation considered is:

$$\ddot{x} + \beta \dot{x} + \omega^2 (1 + p \cos \Omega_d t)x + kx^3 = d \cos(\Omega_d t + \phi),$$

(3)

where $x$ represents the amplifier response, $d$ and $p$ the amplitudes of external and parametric
excitations with frequencies $\Omega_p$ and $\Omega_d$, respectively, $\omega$ the natural frequency, and $k$ the nonline-
arity coefficient (not necessarily small). For a detuned amplifier ($\Omega_p/\Omega_d \neq 2$) the resulting quasi-
periodic response is obtained. It appears that large responses may emerge with arbitrarily small
external excitation, with the amplifier gain tending to infinity.

Finally, to illustrate that the applicability range of the MVA is not restricted to problems with
non-autonomous excitation, self-excited oscillations in autonomous systems are considered for the
Van der Pol equation with strong nonlinearity. As a result stationary as well as non-
stationary responses are determined, and validated numerically.

The considered examples illustrate that the MVA is an efficient tool for treating problems with
strong parametric excitation, combined external and parametric excitation, self-excitation, and
nonlinearity. It may be applied as well for strongly nonlinear systems, though with certain re-
strictions on solution space. [The work is carried out with financial support from the Danish Council for Inde-
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References

Heave-imposed motion in vertical risers: a reduced-order modelling based on Bessel-like modes

Carlos E.N. Mazzilli*, Fabio Rizza#, Thiago Dias*

* Escola Politécnica
University of São Paulo
Av. Professor Luciano Gualberto,
Trav 3 no. 380, 05508-010 São Paulo, Brazil
cenmazzzi@usp.br

# Politecnico di Milano
Piazza Leonardo da Vinci, 32
20133 Milano
Italy
fabio.rizza15@gmail.com

Abstract

Recent discoveries of oil and gas fields off the Brazilian Southeast coast, in ultra-deep waters, pose scientific and technological challenges for safe and economic exploitation. The dynamic analysis of marine risers is of great relevance, due to fatigue of the structural material. This is the case of risers subjected to heave-imposed motions. In fact, heave causes tension modulation along the flexible tubular structure, which may drive Mathieu’s instability [1]. The riser dynamic response is here addressed by a reduced-order model (ROM) of a time-varying prestressed Euler-Bernoulli beam, starting from a continuous formulation via Hamilton’s Principle. The dynamic problem is worked out via a standard Galerkin projection onto Bessel-like modes, as proposed by Mazzilli et al [2]. A SDOF Mathieu-Duffing equation of motion is obtained:

\[ i\dot{h}_h + \eta \dot{h} \alpha_i A_i + \eta \gamma \alpha_j A_j - \left( \alpha_i + \alpha_j \cos \left( \frac{\Omega}{\omega} \tau \right) \right) A_i - \alpha_i A_i + \alpha_j A_j \right\} - \eta^2 \alpha_{e^j} A_{e^j} = 0 , \] (1)

in which over dots mean derivation with respect to time and Morison damping is assumed. The case of the principal parametric instability zone is considered, for which the heave frequency \( \Omega \) is twice the riser first-mode frequency in still water \( \omega_h \). Therefore, the system responds dominantly in the first mode, which validates the hypothesis of a single-degree-of freedom ROM. It should be recalled that due to the non-constant normal force, the riser modes are not sinusoidal, but are better described by Bessel-like shapes [2]. The non-dimensional amplitude \( \eta \) is here defined as the ratio between the mid-span amplitude and the cross-section gyration radius \( r \). Also to be noted, a non-dimensional time has been used in (1), according to \( \tau = \dot{\omega} t \).

Coefficients \( \alpha_i \) to \( \alpha_{e^j} \) refer to system parameters and \( \alpha_{e^j} \) defines the heave excitation:

\[
\begin{align*}
\alpha_i &= \frac{C_i \rho_o r D}{2m(1 + C_o)} \\
\alpha_j &= \frac{\alpha_{e^j} \ell_1}{\ell_1 (1 + C_o)} \\
\alpha_{e^j} &= \frac{E A h_o}{m^2 \ell_1 (1 + C_o)} \\
\alpha_s &= \gamma \frac{\mu r^2}{\ell_1 (1 + C_o)} \\
\alpha_{e^j} &= \frac{C_o^2}{m^2} \\
\alpha_{e^j} &= \frac{E A h_o}{m^2 \ell_1 (1 + C_o)} \\
\alpha_{e^j} &= \frac{E A h_o}{m^2 \ell_1 (1 + C_o)}
\end{align*}
\] (2)

In (2), \( \rho_o \), \( C_i \) and \( C_o \) are, respectively the water density, drag and added-mass coefficients. The riser parameters are: external diameter \( D \), cross-section area \( A \) and moment of inertia \( I \), length \( \ell \), mass \( m \) and immersed weight per unit length \( p \), Young’s modulus \( E \) and static average normal force \( \bar{N} \). The heave motion parameters are its amplitude \( h_h \) and frequency \( \Omega \).
Coefficients $A_i$ to $A_b$ refer to the Galerkin projection onto the normalised Bessel-like first mode $\phi(\xi)$, according to [2]:

$$
A_b = \int_0^1 \phi^2 d\xi \\
A_i = \int_0^1 \phi^2 |\phi|^2 d\xi \\
A_2 = \int_0^1 \phi^{**} \phi d\xi \\
A_i = \int_0^1 \phi^2 d\xi
$$

(3)

Primes denote differentiation with respect to the normalised abscissa $\xi$. ROM’s response (Figure 1) was obtained from numerical integration of (1) for a riser model tested at the São Paulo Technological Research Institute towing tank, using the data of Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell$</td>
<td>2.552m</td>
</tr>
<tr>
<td>$N$</td>
<td>23.65N</td>
</tr>
<tr>
<td>$D$</td>
<td>0.00222m</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>1000kg/m³</td>
</tr>
<tr>
<td>$r$</td>
<td>0.0068m</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.9998</td>
</tr>
<tr>
<td>$m$</td>
<td>1.190kg/m</td>
</tr>
<tr>
<td>$C_d$</td>
<td>2.84</td>
</tr>
<tr>
<td>$p$</td>
<td>7.869N/m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>5.015rad/s</td>
</tr>
<tr>
<td>$EA$</td>
<td>1207N</td>
</tr>
<tr>
<td>$EI$</td>
<td>0.056Nm²</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>10.030rad/s</td>
</tr>
</tbody>
</table>

**Table 1. System parameters**

![Figure 1](image)

**Figure 1.** Time response and phase plane for ROM of a riser under Mathieu’s instability

The maximum non-dimensional amplitude observed was $\eta = 2.2239$, which corresponds to a dimensional amplitude of 0.0151m, in excellent agreement with the experimental result [3].

**References**


Limiting Phase Trajectories as an Alternative to Nonlinear Normal Modes

Leonid I. Manevitch*

* Semenov Institute of Chemical Physics of Russian Academy of Sciences
4, Kosygina str., Moscow, 11991, Russia
manevitchleonid3@gmail.com

Abstract

As it is known, the linear and nonlinear normal vibrations are synchronous single frequency motions of the linear and nonlinear Hamiltonian systems. The corresponding spatial distributions of the displacements are described by linear or nonlinear normal modes (LNMs or NNMs) [1, 2]. They describe also the periodically excited stationary vibrations and self-sustained oscillations in multi-particle non-conservative systems. It was shown recently that in the systems of weakly coupled oscillators the additional class of synchronous solutions can be attributed to the highly non-stationary resonance processes [3-5]. These solutions, which can be adequately described in a slow time scale, were denoted as limiting phase trajectories (LPTs), and a special technique applicable for their study is based on non-smooth temporal transformations [6]. Contrary to LNM and NNM, they describe the maximum possible energy exchange between the oscillators or the clusters of oscillators (effective particles). The conditions of the transition from intensive energy exchange to energy localization can be also formulated in terms of LPTs [4-5].

The concept of LPT is valid in the case when the resonance conditions between different NNMs arise, and therefore the phenomenon of an intermodal coherence takes place. Because of this phenomenon, the resonant NNMs cease to be appropriate tool for dynamical analysis, and transition to effective particles (in 2DoF systems – to real particles) has to be performed. Then, from physical viewpoint one can observe the beats between effective particles each of which is a cluster of the real particles. The beats are described adequately in slow time by LPTs, and corresponding temporal behavior – by non-smooth functions which were introduced earlier for study of vibro-impact or other strongly nonlinear processes [6]. From mathematical viewpoint, the principal difference between NNM and LPT manifests in the basic number systems which are the most suitable in these cases. They are well-known hyperbolic numbers and less known complex (elliptic) numbers and less known hyperbolic numbers, respectively.

When the excitation intensity grows both NNMs and LPTs undergo a series of transformations. For NNMs, they are bifurcations of stationary points in slow time that means the instability of NNM and formation of new stationary points (NNMs), encircled by a separatrix. As for LPT, its coincidence with separatrix leads to a prohibition of complete energy exchange between the effective particles and, as consequence, to the energy localization on the initially excited effective particle.

The applications of LPT concept involve a number of important problems in both mechanics and physics. In particular, they correspond to:

1. Transient vibrations of the forced nonlinear oscillator (in this case LPT describes the most intensive process in which the oscillator takes periodically the maximum possible energy from its source) [7]. If dissipation is taken into account, the LPT concept predicts also the existence of a limit cycle [8] when the excitation includes two harmonic forces with closely spaced frequencies.
2. Nonlinear beats in conservative 2DoF systems corresponding to complete energy exchange in a slow time scale between two weakly coupled oscillators while NNMs are the stationary states. In terms of the LPTs one can predict the transition from complete energy exchange to energy localization on the excited oscillator [9]. In these terms the conditions of efficient targeted energy transfer in the presence of damping are also formulated [10].

3. Intense energy exchange and transition to energy localization in the oscillatory chain containing many degrees of freedom [4].


5. Energy exchange and localization in carbon nanotubes [12].

6. Energy exchange in the systems vibrating in the conditions of acoustic vacuum.

We discuss briefly these problems, opposing in all cases the LPTs and NNMs concepts.

References


Dynamics of a delay limit cycle oscillator with self-feedback

Lauren Lazarus\textsuperscript{\ast}, Matthew Davidow\#, Richard Rand\textsuperscript{\dag}

\textsuperscript{\ast} Dept. Mechanical and Aerospace Engineering
Cornell University
Ithaca, NY 14853, USA
ll479@cornell.edu

\textsuperscript{\#} Dept. Applied and Engineering Physics
Cornell University
Ithaca, NY 14853, USA
mbd83@cornell.edu

\textsuperscript{\dag} Faculty of Dept. Mathematics,
Dept. Mechanical and Aerospace Engineering
Cornell University
Ithaca, NY 14853, USA
rhr2@cornell.edu

Abstract

This paper concerns the various dynamical behaviors of the following nonlinear differential-delay equation (DDE):

\[ \dot{x} = -x(t - T) - x^3 + \alpha x \] (1)

which represents a "delay limit cycle oscillator" with delay \( T \), experiencing self–feedback with coefficient \( \alpha \). This setup was motivated by the system of two coupled delay limit cycle oscillators:

\[ \dot{x} = -x(t - T) - x^3 + \alpha y \] (2)

\[ \dot{y} = -y(t - T) - y^3 + \alpha x \] (3)

On the invariant manifolds defined by the in–phase mode \( x = y \) and out–of–phase mode \( x = -y \), the system is reduced to equation (1), with the self–feedback terms \( +\alpha x \) and \( -\alpha x \) respectively.

The single–oscillator equation (1) is found to have 1 or 3 equilibria depending on the parameter \( \alpha \). In the case of zero delay (\( T = 0 \)), we linearize the equation to find that the \( x = 0 \) equilibrium, which exists for all values of \( \alpha \), changes from stable for \( \alpha < 1 \) to unstable for \( \alpha > 1 \) in a pitchfork bifurcation. This bifurcation creates a pair of stable equilibria \( x = \pm \sqrt{\alpha - 1} \) for \( \alpha \geq 1 \).

The equilibria undergo Hopf bifurcations and change stability at:

\[ T_H = \frac{\arccos \gamma}{\sqrt{1 - \gamma^2}} \] (4)

where \( \gamma = \alpha \) for the \( x = 0 \) equilibrium, and \( \gamma = 3 - 2\alpha \) for the \( x = \pm \sqrt{\alpha - 1} \) equilibria.

The stability and approximate amplitudes of the limit cycles created in each Hopf bifurcation are found by applying the multiple scales perturbation method, as calculated for a generic cubic DDE in [3]. With these results, the limit cycle off of \( x = 0 \) is found to exist and be stable for \( T \geq T_H \). The limit cycles emerging from \( x = \pm \sqrt{\alpha - 1} \) exist for values of \( T \leq T_H \), and are unstable.

These results are corroborated by numerical findings from DDE–BIFTOOL [1], [2]. Additionally, we find that the stable limit cycle emerging from \( x = 0 \) continues to exist for increasing \( \alpha \) until it dies in a limit cycle fold, as shown in Fig. 1. For large delay \( T \to \infty \), this limit cycle appears to approach an approximate square wave, as found by numerical integration, some of whose features we can also explain analytically.
Figure 1: A surface of limit cycles, as compiled from DDE–BIFTOOL results. Limit cycles are born from the origin in a Hopf bifurcation and are destroyed in a limit cycle fold. The locus of the limit cycle fold is shown as both a space curve and its projection onto the $\alpha - T$ plane.

Our study of the delay limit cycle oscillator with self–feedback, eqn. (1), indicates various behaviors based on different regions in $\alpha - T$ parameter plane. These regions can be seen in Fig. 2. (The region for large delay $T$ where we find an approximate square wave solution is not shown.)

Figure 2: Regions of parameter space and the bifurcation curves that bound them.

While we discuss some unstable behaviors, the stable behaviors are of more interest, as they are observable in the overall system. Region I contains a single stable equilibrium; Region V contains a pair of stable equilibria. Regions II and III contain a stable limit cycle. Region IV contains both a stable limit cycle and a pair of stable equilibria. Some of these stable behaviors are observed to also be stable in the full coupled oscillator system, eqns. (2, 3).

References


The problem of rolling without sliding of a heavy rotationally symmetric body on a fixed horizontal plane is a classical problem of nonholonomic mechanics. In 1897, S.A. Chaplygin in his paper [1] proved that the solution of this problem is reduced to the integration of the second-order linear differential equation with respect to the component of the angular velocity of the body in the projection on its axis of symmetry. However, the solution of this differential equation cannot always be found. In the case when the moving body is a nonhomogeneous dynamically symmetric ball, the solution of the corresponding equation is expressed in terms of elementary functions [1]. In the case of motion of a circular disk or a hoop on a horizontal plane, the solution of the corresponding equation is expressed in terms of a hypergeometric series [1]. In the paper [5], Kh.M. Mushtari continued the investigation of the problem of motion of a heavy rotationally symmetric body on a perfectly rough horizontal plane. Under additional condition imposing restrictions on the surface of the rolling body and a mass distribution in it, two new particular cases have been found, when the motion of the body can be investigated completely. In the first case the moving body is bounded by the surface formed by rotating a parabolic arc about an axis passing through its focus, and in the second case the moving body is a rotationally symmetric paraboloid.

In 1986, American mathematician J. Kovacic proposed the algorithm [4] for finding a general solution of a second-order linear differential equation with variable coefficients for a case when this solution can be expressed in terms of so called liouvillian functions [3, 4]. Recall that liouvillian functions are functions that are built up from the rational functions by algebraic operations, taking exponentials and by integration. If a linear differential equation has no liouvillian solutions, the Kovacic algorithm also allows to ascertain that fact. Using the Kovacic algorithm we proved the nonexistence of liouvillian solutions in the problem of motion of a dynamically symmetric thorus on a perfectly rough plane [2].

In this paper we discuss the application of Kovacic algorithm to the problem of motion of a rotationally symmetric ellipsoid on a perfectly rough horizontal plane. Under additional restrictions on the parameters of the system we present several cases when the equations of motion of the ellipsoid can be completely solved in terms of the quadratures. The physical admissibility of these additional restrictions is discussed. Qualitative behaviour of the ellipsoid on the plane is described in the found integrable cases.

References


Asymptotic Analysis of Autoresonant Oscillatory Chains

Agnessa Kovaleva

† Space Research Institute
Russian Academy of Sciences
117997 Moscow, Russia
agnessa_kovaleva@hotmail.com

Abstract

In this paper we study analytically and numerically the emergence of autoresonance (AR) in a multi-dimensional oscillator array. We recall that AR denotes the persistent capture of a nonlinear oscillator into resonance with its drive due to variations of structural and/or excitation parameters. A large number of theoretical, numerical and experimental results concerning AR in different systems have been reported in the literature, see, e.g., [1] and references therein. In most of these studies, AR was considered as an effective method of excitation and control of high-energy oscillations in the entire system. In this work we demonstrate that this conclusion cannot be applied universally, because AR in a multi-dimensional system is a much more complicated phenomenon than AR in a single oscillator, and the behavior of each element in an array may drastically differ from the dynamics of a single oscillator. These effects were recently analyzed for a two-degree-of-freedom cell consisting of weakly coupled linear and nonlinear oscillators [4]. This paper studies a more general problem of energy transfer and localization in a multi-dimensional array consisting of a chain of time-invariant linear oscillators with equal partial frequencies weakly coupled to a nonlinear actuator (the Duffing oscillator) driven by an external force.

Since AR is a purely nonlinear effect, oscillations with growing energy in the linear chain may arise only in the presence of AR in the nonlinear actuator. The objectives of the present work are to find the conditions under which AR in the nonlinear actuator brings about growing oscillations in the coupled chain and define the parameters of the emergent dynamical regimes.

Two types of excitations are considered: (1) an actuator with slowly time-decreasing linear stiffness is driven by a periodic force with constant frequency; (2) the time-independent nonlinear actuator is driven by a force with a slowly-increasing frequency. In both cases, the parameters of the linear chain and linear coupling remain constant, and the system is initially captured into resonance. It was shown earlier that the averaged response and the conditions of the occurrence of AR in a single Duffing oscillator are equivalent for both types of excitation [2, 3]. The derived asymptotic solutions demonstrate that AR in the nonlinear actuator may produce oscillations with increasing amplitudes in the coupled chain only in the system of the first type. On the contrary, in the system of the second type the response of the coupled chain remains bounded despite the presence of AR in the actuator. This means that the systems that seem to be almost identical exhibit different dynamical behavior.

Two distinct types of the dynamical behavior are associated with different resonant properties of the systems. In the system with a constant excitation frequency all oscillators are captured into resonance: the nonlinear oscillator remains captured into resonance due to an increase of the amplitude compensating the change of stiffness, while the partial frequency of the linear oscillators is always close to the excitation frequency. However, in the opposite case, when the
structural parameters remain constant but the forcing frequency slowly increases, AR in the nonlinear oscillator is still sustained by the growth of the amplitude while the attached chain escapes from resonance capture. It is important to underline that the linear chain is actually driven by the gradually increasing coupling response, and the dynamics of the coupled oscillators depends on the relationship between the growth of incoming energy and the loss of energy due to escape from resonance.

Numerical examples demonstrate that exact (numerical) solutions are close to their asymptotic approximations.

References


Analytical and Semi-Analytical Solutions of Some Fundamental Nonlinear Stochastic Differential Equations

Leo Dostal, Edwin J. Kreuzer

Institute of Mechanics and Ocean Engineering
Hamburg University of Technology
Eissendorfer Strasse 42, 21071 Hamburg, Germany
dostal@tuhh.de

Abstract

In the presented work, we use two methods for the determination of analytical and semi-analytical solutions of nonlinear stochastic differential equations (SDE). The first method is based on a limit theorem by Khamsinitski, which is rigorously proven in [3]. From this work, a class of methods was derived known as stochastic averaging. Depending on the analysed nonlinear system and the type of stochastic processes involved, different stochastic averaging methods were developed [4, 6, 2]. The second method that we use was proposed by Pradlwarter [5] and is known as Local Statistical Linearization. This relatively new method is a semi-analytical approach, where the type of stochastic processes involved, different stochastic averaging methods were developed was derived known as stochastic averaging. Depending on the analysed nonlinear system and theorem by Khashminskii, which was rigorously proven in [3]. From this work, a class of methods solutions of nonlinear stochastic differential equations (SDE). The ¿

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Figure 1: Contour lines of Hamiltonian $H_6$.

(2) might be much more difficult to obtain. For such cases, we transform the system (1) into an Itô equation and determine the probability density $p(X,t)$, $X \in \mathbb{R}^n$, $t \in \mathbb{R}$, by a sum of Gaussian densities $p_i$ which yields

$$p(X,t) = \sum_i A_i p_i(X,t), \quad \sum_i A_i = 1, \quad A_i \geq 0.$$  \hspace{1cm} (5)

This is achieved with the Local Statistical Linearization method. The time evolution of mean vectors $\mu_i$ and covariance matrices $C_i$ of each local density $p_i$, are determined by the derivatives of the first and second order moments of $X$. These are obtained from Itô’s differential rule and Gaussian closure.

References


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Analysis of the 1:1 resonant energy exchanges between two coupled nonlinear oscillators

C.-H. Lamarque*, S. Charlemagne*, M. Weiss*, B. Vaurigaud*, A. Ture Savadkoohi*

* LGCB and LTDS UMR CNRS 5513
ENTPE, Université de Lyon
rue Maurice Audin, F-69518 Vaulx en Velin, France
[lamarque, simon.charlemagne, mathieu.weiss, alireza.turesavadkoohi]@entpe.fr
# Département Laboratoire de Bordeaux
Cerema - Direction territoriale Sud-Ouest
24 rue Carton - CS 41635, F-33073 Bordeaux, France
bastien.vaurigaud@cerema.fr

Abstract

Two different coupled oscillators are considered: case I- considers vibratory energy exchanges between a linear main system and coupled NES with local potential; case II-consists of a prestressed main structural system with hardening elasto-plastic behavior which is coupled to a prestressed nonlinear energy sink (NES). System equations of the case I reads:

\[
\begin{align*}
M\ddot{y} + \hat{a}\dot{y} + Ky + \hat{c}(\dot{y} - \dot{x}_1) + \tilde{V}(y - x_1) &= F(\tau) \\
m\ddot{x}_1 + \hat{c}(\dot{x}_1 - \dot{y}) + \tilde{V}(x_1 - y) + \tilde{G}(x_1) &= 0
\end{align*}
\]

Figure 1: Behavior of two coupled oscillators: the main system linear behavior+NES with cubic global and local potentials: a) invariant manifold at fast time scale and exact behavior obtained by direct numerical integration. \(N_1\) and \(N_2\) stand for amplitudes of the main oscillator and the NES, respectively; b) exact time history of the energy of the NES with cubic global and local potentials; c) exact time history of the energy of main linear oscillator.

steps are followed: 1) to shift the system to the center of mass and relative displacements; 2) complexification of the system by applying variables of Manevitch [1]; 2) using Galerkin’s technique: keeping first harmonics and truncating higher ones; 3) to embed the time \(\tau\) to fast, \(\tau_0 = \tau\), and
slow $\tau_1 = \varepsilon \tau$ scales. 4) using a multi-scale method and analyzing the system at fast time scale (to obtain invariant manifold) and slow time scale (reducing the order of the system to 2: searching for equilibrium point, periodic regimes, and singular ones, strongly modulated responses). A typical invariant manifold of such a system with corresponding numerical results and histories of amplitudes of two oscillators are given in Fig. 1. It can be seen that for this example the system faces strongly modulated response [2]. For analyzing the case II, the linear behavior of the main systems in Eq. 1 should be replaced by equations representing elasto-plastic behavior and we assume that the NES does not posses any local potential i.e. $\tilde{G}(x_1) = 0$. Here, we use two different methods for treating system equations: a) exact solution of the system is constructed piece-by-piece by time-event-driven exact analytical method and then b) we implement the multi-scale technique which is explained in case I. Multi-scale technique leads us to have a very strong analytical tools for tuning NES systems according to the design goal which can be passive control of main structural systems and/or harvesting their vibratory energy. The behavior of the designed system can be confirmed by exact solutions (time-event-driven exact analytical technique). A typical invariant manifold of such a system under prestressing terms and corresponding numerical results are presented in Fig. 2. As the previous case, here we can see the strongly modulated response of the system which is due to the existence of singular points at the slow time scale [2].

Figure 2: Behavior of two coupled oscillators: the main system with hardening elasto-plastic behavior+NES with piece-wise linear potential: a) invariant manifold at fast time scale and exact behavior obtained by time-event-driven method. $N_1$ and $N_2$ stand for amplitudes of the main oscillator and the NES, respectively. The system presents strongly modulated response between second pair of stability borders which correspond to higher energy levels; b) exact time history of the energy of the NES with piece-wise linear potential; c) exact time history of the energy of main oscillator with elasto-plastic behavior

References


Evaluation of a Kinematic Approach for Backward Whirl with an Application to Drillstring Dynamics

Westermann. Henrik*, Wallaschek. Jörg*

* Institute of Dynamics and Vibration Research
Leibniz University of Hannover
Appelstraße 11, 30167 Hannover, Germany
[westermann, wallaschek]@ids.uni-hannover.de

Abstract

Drillstring vibrations are one of the major reasons for tool failure-related non-productive times in a deep drilling operation. A drillstring used for oilwell drilling is an extremely slender structure and exhibits different types of vibrations. Lateral dynamics in the bottom hole assembly (BHA) are an important cause of premature failure of drillstring components and drilling inefficiency. The whirling motion of drill collars has been widely reported in the literature. The current state of the art in drilling dynamics analysis is finite element modeling (FEM) with a large number of unknowns and very long computation times.

One of the current topics in the oil and gas industry is the automation of the drilling process. Real-time models are currently evaluated to simulate lateral BHA vibrations. In the literature the Jeffcott rotor with contact is the most used model (cf. [1], [2] and [3]). The equations of motion (EOM) for the basic Jeffcott rotor are

\[
\begin{align*}
mx'' + bx' + kx + \Theta k_W \left(1 - \frac{c_0}{r}\right)(x - \mu(v_{rel})y) &= me_0\Omega^2 \cos(\Omega t), \\
m\dot{y} + b\dot{y} + ky + \Theta k_W \left(1 - \frac{c_0}{r}\right)(\mu(v_{rel})x + y) &= me_0\omega^2 \sin(\Omega t),
\end{align*}
\]

where \(m\) is the modal mass, \(b\) is the modal damping and \(k\) is the modal stiffness. The Heaviside function \(\Theta\) is zero, if the radial displacement \(r\) is smaller than the clearance \(c_0\) between drillstring and borehole. Otherwise, \(\Theta\) is equal to one. The contact is modeled with a penalty stiffness \(k_W\) and a discontinuous Coulomb friction model. The friction function is

\[
\mu(v_{rel}) = \begin{cases} 
-\mu, & \text{for } v_{rel} \leq 0, \\
\mu, & \text{for } v_{rel} \geq 0,
\end{cases}
\]

where \(v_{rel}\) is the relative velocity between drillstring and borehole and \(\mu\) is the friction factor. The system is excited with an eccentricity \(e_0\) and driven with the rotation speed \(\Omega\). The time is denoted with \(t\) in Equation (1). The weight on bit (WOB), which is applied at the bit, leads to an axial force along the drillstring. Here this is assumed as a pure static compression force \(W_0\) and it can be considered in the stiffness coefficient.

The model exhibits different vibration phenomena including separation, partial contact, forward whirl, backward whirl, backward whip and chaos. The vibration types are analyzed in [4] and [5] with time and frequency domain approaches. For drilling applications, only separation, forward and backward whirl are reported. Backward whirl is particularly important because it is the most harmful vibration phenomena.
The analysis of backward whirl is difficult because the EOM is highly nonlinear due to the Coulomb friction characteristic. Separation and forward whirl have been analyzed in [6] using a path-following technique, which is based on the harmonic balance method (HBM) in combination with the asymptotic-numerical method (ANM). This continuation technique is proposed in [7].

The aim of this paper is to evaluate a kinematic approximation of the model results in case of backward whirl. The normal contact force to the well bore can be expressed with the algebraic equation

\[ F_N = -kc_o + mc_0 \left( \frac{R_C}{c_0} \right)^2 + me_0 \Omega^2 \cos \left( \left( 1 + \frac{R_C}{c_0} \right) \Omega t \right) \]  

(3)

where \( R_C \) is the radius of the drillstring. As long as the normal contact force is positive backward whirl is stable. If \( F_N \) becomes negative backward whirl disappears. Due to this assumption the lower boarder between backward whirl and separation can be found. Equation (3) can be solved without numerical integration in a minimum amount of computation time. In addition, stability maps are calculated that show the vibration states separation and backward whirl over the input parameters \( W_0 \) and drillstring rotations per minute (RPM). The stability maps are compared with numerical model results. The reference drillstring section has been used by an oil and gas service in a field test. The evaluated algorithm is real-time capable and a possible approach to design a drilling advisory system for decreasing lateral BHA vibrations.

References


Phase Bifurcations in an electromechanical system

Márcio José Horta Dantas∗, Rubens Sampaio#, Roberta Lima†

∗ Faculdade de Matemática
Universidade Federal de Uberlândia
38400-902 Uberlândia, MG, Brazil
marcio.horta.dantas@gmail.com

# Mechanical Engineering Department,
PUC-Rio,
22451-900 Rio de Janeiro, RJ, Brazil
rsampaio@puc-rio.br †roberta_10_lima@hotmail.com

Abstract

Consider the following electromechanical system

![Cart Motor System](image1)

![Circuit with a time-dependent source](image2)

The equations of motion of the system given in Figures ?? and ?? are the following ones

\[ l c' (t) + k_e \alpha' (t) + r c (t) = \nu + \chi \sin (\omega_1 t), \]

\[ \left( d^2 \sin (\alpha (t))^2 + b + b_m \right) \alpha' (t) + \left( d^2 m \sin (\alpha (t))^2 + j_m \right) \alpha'' (t) \]

\[ + d^2 m \cos (\alpha (t)) \sin (\alpha (t)) \alpha' (t)^2 - k d^2 \cos (\alpha (t)) \sin (\alpha (t)) - c (t) k_t = 0. \]

where \( r \) is the time, \( \nu \) and \( \chi \) are constant voltages, \( c \) is the electric current, \( \alpha' \) is the angular speed of the motor, \( l \) is the electric inductance, \( j_m \) is the motor moment of inertia, \( b_m \) is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, \( k_t \) is the torque constant, \( k_e \) is the motor electromagnetic force constant and \( r \) is the electrical resistance. On this cart is applied an elastic force \(-k x\) and a viscous damping given by \(-b x'\).

The deduction of (??) can be obtained from the first principles as it was done in [?]. Let us write (??) in dimensionless form. Consider the following dimensionless parameters and functions given by

\[ s = \frac{r}{\omega_1}, p(s) = \alpha \left( \frac{l s}{r} \right), q(s) = \frac{1}{r} \alpha' \left( \frac{l s}{r} \right), w(s) = \frac{1}{k_e} \left( \frac{l s}{r} \right), \alpha = \frac{\omega_1 l}{r}, \]

\[ v_0 = \frac{\nu l}{k_e r}, v_1 = \frac{d^2 m}{j_m}, v_2 = \frac{k_e l k_t}{j_m r^2}, v_3 = \frac{b_m l}{j_m r}, v_4 = \frac{\chi l}{k_e r}, v_5 = \frac{l b}{m r}, v_6 = \frac{k d^2}{m r^2}. \]

By performing the change of scale \( s \rightarrow s / \omega \) and taking \( \epsilon = v_1 \), just to emphasize \( v_1 \) is the small parameter, one obtains from (??)

\[ p' (s) = q(s) / \omega, \]

\[ w' (s) = \left( v_4 \sin (s) - w(s) - q(s) + v_0 \right) / \omega, \]

\[ q' (s) = \left( -\epsilon v_5 q(s) \sin^2 (p(s)) - \left( \epsilon q(s)^2 - \epsilon v_6 \right) \cos (p(s)) \sin (p(s)) \right) \]

\[ + v_2 w(s) - v_3 q(s) \left( \omega \epsilon \sin^2 (p(s)) + 1 \right). \]
Now we want to put the system in a convenient form to use results found in the literature. After a long sequence of non-trivial changes of variables, (10)-(12) can be rewritten as

\[ p_1'(s) = \varepsilon (z_1(s) + F_2(s, p_1(s))) / B_0, \quad y'(s) = Ay(s) + \varepsilon \tilde{F}_3(s, p_1(s), y(s), \varepsilon), \] (5)

where \( y = (w_1, z_1) \), \( B_0 = v_0 v_2 / (v_3 + v_2) \), \( A \) is a 2 \times 2 matrix which entries \( A_{ij} \) are the following ones: \( A_{11} = A_{12} = -1 / B_0, A_{21} = v_2 / B_0, A_{22} = -v_3 / B_0. \) Moreover, for each parameter \( u \) let \( F(s, u) = (F_1(s, u), F_2(s, u)) \) be the unique 2\( \pi \) periodic solution of \( \frac{partial F}{partial s} = AF(s, u) + h_2(s, u). \) And, \( h_2 = (0, K_1 \cos(K_4 \cos(s + \phi_1) + s + 2p_1 + \phi_1) + K_2 \cos(s + \phi_2) + K_3) \), where \( K_1, K_2, K_3, \phi_1, \phi_2, \phi_3 \) are complicated functions of the parameters \( v_i. \) The variable \( p_1 \) means the phase associated to (10)-(12) and \( w_1, z_1 \) stand for the current and angular speed respectively. The \( \tilde{F}_3 \) mapping is 2\( \pi \) periodic in the variable \( s. \) Since (10)-(12) was rewritten in the form given by (10), one is ready to use the Theorem 3.4 of [?, pg.196] in order to investigate the existence and stability of periodic solutions of the last system. So, it is only necessary to take into account the average of the first equation of (10). Let us consider the Fourier expansions \( \cos(\beta \cos(s)) = J_0(\beta) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(\beta) \cos(2ns) \), \( \sin(\beta \cos(s)) = 2 \sum_{n=0}^{\infty} (-1)^n J_{2n+1}(\beta) \cos(2n+1)s \) where \( J_l \) are Bessel function of the first kind. From these equations, the averaged equation can be simplified and one obtains that

\[ p_1' = \varepsilon ((K_3 / (v_3 + v_2)) - (K_1 J_1(K_4) / (v_3 + v_2)) \sin(2p_1 + \phi_1 - \phi_3)), \] (6)

where \( J_1 \) is the Bessel function of the first kind of order 1. Let us take \( \varepsilon = K_3 / (K_1 J_1(K_4)) \) and assume the condition \( |\varepsilon| < 1 \) in order to ensure the existence of equilibrium points for (10). It can be shown that (10) always has four hyperbolic equilibrium points, two stable and two unstable ones. From the above mentioned theorem this leads to four 2\( \pi \) periodic orbits with the same type of stability. Moreover, if \( K_3 \neq 0 \) the stability of the equilibrium points, and hence of the associated periodic orbits, does not change when \( K_4 \) varies along \( \mathbb{R}^+. \) Of course it is assumed that \( K_4 \) is different from each positive zero of \( J_1. \) When \( K_3 = 0 \) one gets bifurcations of those periodic orbits. Since \( K_3 = -v_0 v_2 v_3 / (v_3 + v_2), \) it is worth to note the condition \( K_3 = 0 \) is equivalent to \( v_3 = 0, \) or \( b = 0 \) i.e. absence of viscous damping. This leads to the following bifurcation phenomena: when \( v_3 \) or \( v_4 \) varies in \( \mathbb{R}^+ \) there is a sequence of consecutive open intervals \( I_i, \) whose extremities depend on the zeros of \( J_1, \) such that if an equilibrium point of (10) is stable/unstable when \( K_4 \in I_i \) then this same equilibrium point is unstable/stable when \( K_4 \in I_{i+1}. \) With this result, if one takes three consecutive intervals such that in the first one the equilibrium point is stable, in the second one has instability and in the third stability again. This is called the Sommerfeld effect as it was defined in [?, Subsection 6.1]. In this case the Sommerfeld effect is interpreted as a bifurcation of phase, that seems to be a new and interesting case.

References


Knowledge-based Analytical Approaches to Characterization of Self-excited Oscillations in Rotor-Stator Rubbing Systems

Jun Jiang*, Ling Hong#, Yanhua Chen†

*State Key Lab. of Strength and Vib.  Xi’an Jiaotong University
Xi’an 710049, China  hongling@mail.xjtu.edu.cn
jun.jiang@mail.xjtu.edu.cn

#State Key Lab. of Strength and Vib.  Xi’an Jiaotong University
Xi’an 710049, China  hongling@mail.xjtu.edu.cn

†Center of Intelligent Control and Telescience
Graduate school at shenzhen, Tsinghua University
The University Town, Shenzhen 518055, P.R.China
chy0623@126.com

Abstract

The dynamics of engineering systems, which are generally governed by nonlinear, sometimes also non-smooth, differential equations, are very complicated. The rotor-to-stator contact rubbing, modeled by multiple DOF non-smooth nonlinear systems, is one of such examples and of great practical interests. Thus there exist results of extensive simulations and many valuable experiments in the literature as mentioned in [1,2] and the references therein. However, the progress in determining the existence and the onset boundaries as well as the whirl frequencies of self-excited oscillations with the variation of system parameters in the rotor-stator rubbing systems is not satisfactory. For instance, when the elasticity on the contact surfaces is taken into account, both experimental and computational results show that multiple self-excited backward whirl motions with different whirl frequencies may coexist in some rotating speed ranges[2], which has never been predicted from the approach given in [3].

In this work the knowledge gained from the existing numerical and experimental results on the generic mechanism and the intrinsic characteristics of the self-excited oscillations in rotor-stator rubbing systems are sufficiently utilized to make the classical Harmonic Balance method and Multiple Scale method very effective in obtaining the analytical existence conditions and the semi-analytical onset conditions of self-excited oscillations in a 4-DOF rotor-stator rubbing model, which includes both the elasticity at the contact surfaces and the cross-coupling effects. Since the mechanism of self-excitation for both backward and forward whirls are introduced in the presented model (see Eq.(1)), the system can undertake a self-excited forward whirl motion due to the cross-coupling effects, beside the traditional self-excited dry friction backward whirl motions. Furthermore, we also demonstrate that the analytically solved Nonlinear Normal Modes (NNMs) of the non-smooth nonlinear system will provide very useful information in deriving and characterizing the critical conditions of the self-excited oscillations.

The governing equations of the rotor-stator rubbing system are given in the following form:

\[ m \ddot{r}_r + \left( c_r - j \omega_r \right) \dot{r}_r + \left( k_r - j \omega_r \right) r_r + \Theta F = m \epsilon \omega^2 e^{j \omega t}, \]

\[ m \ddot{r}_s + c_s \dot{r}_s + k_s r_s - \Theta F = 0, \]

with \( F = k_c \left( 1 + j \mu \cdot \text{sgn}(v_{\text{rel}}) \right) \left( r_r - r_s - \delta \frac{r_r - r_s}{r_r - r_s} \right), \quad v_{\text{rel}} = r_{\text{disk}} \omega + |r_r - r_s| \omega_m, \]

Equation (1)
where \( r_r = x_r + jy_r \) and \( r_s = x_s + jy_s \) are respectively the complex deflections of the rotor and the stator, \( m_r \) and \( m_s \) are the masses, \( c_r \) and \( c_s \) the damping constants, \( k_r \) and \( k_s \) the stiffness. \( \omega_r \) is the rotating speed, \( \delta \) the clearance between the rotor and the stator, \( k_c \) is the stiffness of elastic contact surfaces, \( \nu_{rel} \) is the relative velocity at the contact point with \( \omega_{nk} \) being the whirl frequency and \( r_{disk} \) the radius of the rotor. The friction coefficient is denoted by \( \mu \) and resultant contact force by \( F \). The cross-coupling effects include both damping term \( \gamma_r \) and stiffness term \( Q_r \) for the rotor. Additionally, we define \( \Theta=1 \), if \( |r_r - r_s| \geq \delta \) and \( \Theta = 0 \), if \( |r_r - r_s| < \delta \).

Through the analysis of the proposed analytical approaches, the following conclusions can be drawn. The self-excited forward whirl motion exists just in the parameter regions where the NNM with positive modal frequency is stable. The onset boundary for the self-excited backward whirl can be determined from a perturbation analysis near the modal frequencies of the NNM with negative model frequencies. There are two existence regions for the self-excited backward whirl motions, either isolately standing, or overlapping partly, or one containing the other completely, depending upon the system parameters. From analysis of the Nonlinear Normal Modes (NNMs), we can predict the necessary conditions or the lower limits for the existence of the self-excited backward whirl motions and the lower and the upper limits of the corresponding backward whirl frequencies. This information will benefit engineering designs.

As examples, Fig.1a shows the existence boundaries of two self-excited backward whirl motions that coexist in the dashed overlapped region. Fig.1b presents whirl frequencies vs. rotating speed for a given friction coefficient. Jump phenomenon can be detected with increase and decrease of the rotating speed that well corresponds with test results as mentioned in [2].

![Figure 1](image)

**Figure 1.** (a) Existence boundaries (solid curves) of two self-excited backward whirl motions and their necessary conditions (dashed lines) derived from NNMs; (b) The corresponding backward whirl frequencies (solid curves) when \( \mu=0.2 \) and their upper limits (dashed lines) and lower limits (circles) obtained from the analysis of modal frequencies of NNMs.

**References**


Stationary and transient resonant response of spring pendulum

Jan Awrejcewicz*, Roman Starosta#, Grażyna Sypniewska-Kamińska#

* Department of Automatics and Biomechanics
Technical University of Łódź
Stefanowskiego 1/15, 90-924 Łódź, Poland
awrejcew@p.lodz.pl

# Institute of Applied Mechanics
Poznań University of Technology
Jana Pawła II 24 , 60-965 Poznań, Poland
[roman.starosta, grazyna.sypniewska-

kaminska]@put.poznan.pl

Abstract

Behavior in resonant conditions is one of the most important factors that must be taken into account in design of any devices and machine parts. Analytical investigations, based on various approximate methods, are relatively well developed for the case of the stationary resonance response. In the papers [1, 2, 3] the method of limiting phase trajectories (LPTs) is proposed and successfully applied in analytical approach to the transient resonance response of 1 degree of freedom system as well as to weakly coupled oscillators. The present paper deals with, the dynamic response of the damped pendulum with nonlinear spring. Two regimes of resonant vibration, stationary and non-stationary, are considered. The multiple scales method (MSM) in time and the idea of LPTs were adopted in the analysis. The material point of mass m which is suspended from a fixed point via nonlinear and massless spring is considered. L is the equilibrium length of the spring. The relationship between the elastic force \( F_t \) and the spring elongation \( \Delta l_s \) reads as
\[
F_t = k_1 \Delta l_s + k_2 \Delta l_s^3.
\]
The force \( F_t \cos(\Omega t) \) acts along the spring, and the force \( F_t \cos(\Omega t) \) is perpendicular to the spring axis. The damping in longitudinal as well as in transverse direction is assumed as viscous.

The equations of motion of the system written in the dimensionless form are as follows:
\[
\ddot{z} + c_1 \dot{z} + z + \alpha \dot{z}^3 + 3\alpha x^2 z + 3\alpha x_0^2 z + w^2 (1 - \cos\phi - (z + 1)\dot{\phi}^2 = f_1 \cos(\Omega_1 t),
\]
\[
(z + 1)(\dot{\phi} + \omega^2 \sin \phi + \dot{\phi} + \dot{\phi}_0 + 2z) = (z + 1) \cos(\Omega_2 t),
\]
where, \( \omega_0 = \sqrt{k_1 / m}, \omega_2 = \sqrt{g / L} \), \( \tau = t \omega_0 \) is dimensionless time, \( z(\tau) \) and \( \phi(\tau) \) are generalized coordinates, \( \alpha = k_2 / k_1 \), \( f_1 = F_t / (L \omega_0^2) \), \( f_2 = F_t / (L \omega_0^2) \), \( \omega = \omega_2 / \omega_0 \), \( \Omega_1 = \omega_1 / \omega_0 \), \( \Omega_2 = \omega_2 / \omega_0 \), \( c_1 \) and \( c_2 \) are dimensionless damping coefficients, \( z_0 \) denotes the equilibrium elongation of the spring which fulfills the equation \( \alpha z_0^3 + z_0 = w^2 \).

The equations (1) – (2) are supplemented by the adequate initial conditions. The introduction of the phase space coordinates reduces the order of equations (1)-(2) by one. The functions \( \psi_{\alpha}(\tau) = (\dot{z}(\tau) + iz(\tau))e^{-i\tau} \) and \( \psi_{\phi}(\tau) = (\dot{\phi}(\tau) / w + \phi(\tau))e^{-iw\tau} \) allow to extract the terms responsible for vibration with natural frequencies and their higher harmonics. In order to obtain the resonance response the MSM is used. The following assumptions for magnitude of the quantities affecting the vibration
\[
c_1 = \tilde{c}_1 \varepsilon, c_2 = \tilde{c}_2 \varepsilon^2, z_0 = \tilde{z}_0 \varepsilon, \quad f_1 = \tilde{f}_1 \varepsilon, f_2 = \tilde{f}_2 \varepsilon^2,
\]
are made, where \( \varepsilon \) is a small parameter.

Adopting three time scales, the solutions are searched in the following form
\[
\psi_{\alpha}(\tau; \varepsilon) = \sum_{k=1}^{k=3} \varepsilon^k \xi_{\alpha k}(\tau_0, \tau_1, \tau_2) + O(\varepsilon^4), \quad \psi_{\phi}(\tau; \varepsilon) = \sum_{k=1}^{k=3} \varepsilon^k \xi_{\phi k}(\tau_0, \tau_1, \tau_2) + O(\varepsilon^4).
\]
Afterwards, the case of the main resonance $p_2 \approx w$ and $p_1 \approx 1$ is analyzed. In order to deal with this case the detuning parameters $\sigma_1 = \tilde{\sigma}_1 e^2$ and $\sigma_2 = \tilde{\sigma}_2 e^2$ have been introduced

$$p_1 = 1 + \sigma_1 \quad \text{and} \quad p_2 = w + \sigma_2.$$  

Finally, the following four modulation equations

$$\frac{da_1}{d\tau} = -\frac{1}{2} c_1 a_1 + \frac{1}{2} f_1 \cos \theta_1,$$  

$$a_1 \frac{d\theta_1}{d\tau} = -\frac{3}{2} c_2 a_1 + \sigma_1 a_1 - \frac{3}{8} a_1^3 + \frac{3w^2(w^2-1)}{4-16w^2} a_1 a_2 + \frac{1}{2} f_1 \sin \theta_1,$$  

$$\frac{da_2}{d\tau} = -\frac{1}{2} c_2 a_2 + \frac{1}{2} f_2 \cos \theta_2,$$  

$$a_2 \frac{d\theta_2}{d\tau} = \sigma_2 a_2 + \frac{3w(w^2-1)}{4-16w^2} a_2 a_1^2 + \frac{w(8w^4-7w^2-1)}{64w^2-16} a_2^3 - \frac{1}{2w} f_2 \sin \theta_2,$$  

are derived, where $a_1, a_2$ are amplitudes, and $\theta_1, \theta_2$ are modified phases. The system (6) – (9) is autonomous which enables to analyze the stationary and non-stationary vibration. The stationary resonance response curves and amplitude modulation connected with the non-stationary case of resonance, obtained for the following values: $\alpha = 0.4$, $f_1 = 0.0007$, $f_2 = 0.0005$, $c_1 = 0.002$, $c_2 = 0.00025$, $w = 0.275$, are presented in Fig 1.

![Resonant curves and amplitude modulation in the case of main resonance](image_url)

**Figure 1.** Resonant curves and amplitude modulation in the case of main resonance.

The concept of LPTs allows to investigate the non-stationary processes in the quasi-linear as well as strongly non-linear regimes. The main advantage of this approach consist in possibility to study the relationship between amplitude and phase both in the stationary and non-stationary cases.

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**References**


New Resonances and Velocity Jumps in Nonlinear Road-Vehicle Dynamics

Walter V. Wedig  
KIT- University of Karlsruhe, Germany, E-mail: wwedig@t-online.de

Vehicles riding on uneven rough road surfaces are excited to vertical vibrations described by linear system equations provided the vehicle velocity is assumed to be constant. The associated solutions determine the process characteristics of the stationary driving force needed to control and maintain a constant velocity of the vehicle. More important is the inverse problem of a constant driving force meanwhile the velocity process is stationary and fluctuates as the consequence of the random up and down of road surfaces. The paper investigates second order road profiles modeled by linear filter equations under white noise which allow to pass to the limiting profile of harmonic wave roads. Note that both, the forward and backward drive, are possible. Because of the stationary velocity fluctuations, the describing equations of motion are highly non-linear effecting velocity jumps when the driving force reaches critical values. These investigations are extended to the control of multi-body vehicle systems on road. For sweeps of in- or decreasing driving forces, several velocity jumps are created with considerably higher or lower levels of kinetic energy.

Figure 1: Quarter car on random rough roads  
Figure 2: Simulated 2nd order road spectra

To explain new resonances and jump phenomena in detail, let us consider the vertical and horizontal vibrations of the quarter car, shown Fig.1, described by the two equations of motion

\[
\begin{align*}
\dot{Y}_t + 2D\omega^2_t(\dot{Y}_t - \ddot{Z}_t) + \omega^2_t(Y_t - Z_t) &= 0, \quad g, \mu = 0, \\
\dot{V}_t &= \frac{1}{m}F_t + [2D\omega^2_t(\dot{Y}_t - \ddot{Z}_t) + \omega^2_t(Y_t - Z_t)] \frac{\ddot{Z}_t}{V_t},
\end{align*}
\]

where \( Z_t \) denotes the vertical road profile process generated from white noise \( \dot{W}_t \) by the equations

\[
\begin{align*}
\dot{Z}_t &= \Omega V_t U_t, \\
\dot{U}_t &= -2\delta\Omega|V_t|U_t - \Omega V_t Z_t + \sigma\sqrt{|V_t|}W_t.
\end{align*}
\]

The second order filter equations (3) are linear and lead to stationary solutions with the rms-values \( \sigma^2 = \sigma^2_u = \sigma^2/(4\delta\Omega) \) independent on the vehicle-velocity \( V_t \). The associated road spectra in Fig. 2 are obtained by stochastic Taylor integration schemes and FFT- routines for two way frequencies \( \Omega \) and dimensionless decay factors \( \delta \) applying any constant velocity \( V_t \equiv \pm 1 \) and noise intensity \( \sigma = 1 \). Assuming, that gravity \( g \) and dry friction \( \mu \) are negligible, the linear analysis is completed by calculating the expectation \( E(F_t) \) of the driving force necessary to maintain a constant velocity.

\[
\frac{E(F_t)}{c\sigma^2} = \frac{Dv^2}{2\delta} \frac{v^3 + 4\delta v(\delta + Dv)}{(1 - v^2)^2 + 4v(D + \delta v)(\delta + Dv)}.
\]
In Fig. 3, the mean force (4) is plotted vs the related car velocity \( v = \frac{V_t}{\omega_1} \). More complicated is the inverse problem that the driving force \( f \) is constant and the velocity process \( \dot{V}_t \) has to be calculated applying the equations of motion and now highly non-linear filter equations (3). In Fig. 3, simulation results, obtained by means of bi-linear Euler schemes, are marked by red and yellow circles for forward and backward f-sweeps, respectively. Fig. 4 shows corresponding results for the special case of harmonic wave roads with amplitude \( \sigma_z = z_0 \) and bandwidth \( \delta = 0 \). In both cases, perturbation methods are applied to determine critical force-velocities of jumps, analytically.

![Figure 3: Stochastic velocity jumps of cars](image1.png)

![Figure 4: Speed jumps of cars on wave roads](image2.png)

These investigations are extended to multi-body road-vehicles described by the vector equation

\[
\begin{aligned}
\dot{\tilde{Y}}_t &= A\tilde{Y}_t + \tilde{\sigma}_p W_{t+T/2} + \tilde{\sigma}_m W_{t-T/2}, \\
\dot{Z}_t &= -|V_t|\Omega Z_t + \sigma \sqrt{|V_t|} W_t, \\
AE(\tilde{Y}_t'\dot{\tilde{Y}}_t') + E(\dot{Y}_t'\dot{\tilde{Y}}_t')A' + \tilde{\sigma}_p \tilde{\sigma}_p' + \tilde{\sigma}_m \tilde{\sigma}_m' + e^{TA}\tilde{\sigma}_p \tilde{\sigma}_p' + \tilde{\sigma}_m \tilde{\sigma}_m' e^{TA}' = 0,
\end{aligned}
\]

where \( T \) is the time difference between front and rear wheels and \( Z_t \) is the base excitation through the road profile generated from white noise \( W_t \) by means of a first order filter (5). To give a first application, consider the linear road-vehicle system with four degrees of freedom, shown in Fig. 5. Provided that the velocity \( V_t \) is constant, the stationary responses are investigated by means of the extended covariance matrix equation (6). Fig. 6 shows four related root-mean-squares plotted versus the speed parameter \( \frac{V_t}{\Omega} \) applying two different linear scales in the first two speed ranges and a hyperbolic scale in the last one. Interrupted and solid lines represent rms-results for a very small and extremely long axle distance, respectively. Colored lines with black central ones belong to the realistic axle distance \( \alpha \Omega = 5 \). Obviously, there are main rms-resonances when the speed-frequency of the car approaches its natural frequencies. Additionally, sub-resonances are observable in the low speed range due to the delayed and anticipating excitations through the rear and front wheels.

![Figure 5: Vehicle riding on random road](image3.png)

![Figure 6: New resonances for growing speed](image4.png)
Discontinuity Induced Bifurcations in Nonlinear Systems


* Research Scholar, # Professor
Machine Design Section, Department of Mechanical Engineering
Indian Institute of Technology Madras, Chennai, 600036, India
santhoshbpillai@gmail.com, narayans@iitm.ac.in, mouli@iitm.ac.in

Abstract

Nonlinear systems involving impact, friction, free-play, switching etc. are discontinuous and exhibit sliding and grazing bifurcations when periodic trajectories interact with the discontinuity surface which are classified into crossing sliding, grazing sliding, adding sliding and switching sliding bifurcations depending on the nature of the bifurcating solutions from the sliding surface. The sudden onset of chaos and the stick-slip motion can be explained in terms of these bifurcations. This paper presents numerical and numerical-analytical methods of studying the dynamics of harmonically excited systems with discontinuous nonlinearities representing them as Filippov systems. The switch model based numerical integration schemes in combination with the time domain shooting method are adopted to obtain the periodic solutions and the bifurcations.

The Filippov system is represented in terms of the smooth vector fields as \( \dot{x} = F_1(x, \alpha, t) \) for \( x \in S_1 = \{ x : H(x) > 0 \} \) and \( \dot{x} = F_2(x, \alpha, t) \) for \( x \in S_2 = \{ x : H(x) < 0 \} \) with the boundary between the smooth regions \( S_1 \) and \( S_2 \) given by \( \Sigma = \{ x : H(x) = 0 \} \) where \( H(x) \) is a scalar function representing the discontinuity surface and \( \alpha \) is a parameter.

The system of a mass on a moving belt [1, 2] given by equation (1) rewritten as equation (2) is considered as an example.

\[
\begin{align*}
 mx - c_1(\dot{x} - v) + c_2(\dot{x} - v)^3 + k_1 x + k_2 x^3 + \mu N \text{sgn}(\dot{x} - v) &= F_0 \cos \omega t \quad (1) \\
 \dot{x} - \alpha g(\dot{x} - v) + \beta g(\dot{x} - v)^3 + \gamma_1 x + \gamma_2 x^3 + \mu g \text{sgn}(\dot{x} - v) &= f_0 \cos \omega t \quad (2)
\end{align*}
\]

where \( \alpha = \frac{c_1}{mg}, \beta = \frac{c_2}{mg}, \gamma_1 = \frac{k_1}{m}, \gamma_2 = \frac{k_2}{m} \) and \( f_0 = \frac{F_0}{m} \). The discontinuity surface is given by \( H(x) = \dot{x} - v \). Following parameter values \( f_0 = 10 m/s^2, g = 9.81 m/s^2, \mu = 0.6, \alpha = -0.05 s/m, \beta = 0, \gamma_1 = 0, \gamma_2 = 10000 m/s^2 \) and \( v = 1 m/s \) are considered. The bifurcation diagrams with \( \omega \) as the parameter in two frequency ranges obtained by the switch model numerical integration are shown in Figure 1.

Figure 1: (a) period doubling route to chaos (b) sudden onset of chaos.
figures 1(a) and 1(b) which show respectively a period doubling route and a sudden transition to chaos. The phase planes of the periodic and chaotic solutions along with the Poincare sections are shown in Fig. 2. The discontinuity induced bifurcations (DIB) are identified and verified using the following conditions of sliding bifurcations [3].

\[ \nabla H(x^*) = 0; \quad \nabla H(x^*) \neq 0; \quad \left( \nabla H, F_1^* \right) \big|_{(x^*, t^*)} = 0, \]

where \((x^*, t^*)\) is the bifurcation point and \(\nabla\) is the gradient operator. Other conditions for the bifurcations are

\[ \left( \nabla H, \frac{\partial F_1}{\partial x} \right) \big|_{(x^*, t^*)} > 0 \] (crossing sliding and grazing sliding),

\[ \left( \nabla H, \frac{\partial F_1}{\partial x} \right) \big|_{(x^*, t^*)} < 0 \] (switching sliding) and

\[ \left( \nabla H, \frac{\partial^2 F_1}{\partial x^2} \right) \big|_{(x^*, t^*)} = 0 \] and

\[ \left( \nabla H, \left( \frac{\partial F_1}{\partial x} \right)^2 \right) \big|_{(x^*, t^*)} < 0 \] (adding sliding) are also verified. Lyapunov exponents are calculated by chaos synchronization method applied to the Filippov systems to confirm chaotic motion. In this method, the augmented synchronous system also has discontinuous nonlinearities making the combined system a two dimensional Filippov system adding a dimension of difficulty [4]. The combined two dimensional Filippov system is numerically integrated using the switch model taking into consideration the sliding along either of the discontinuous surfaces or along the intersection of the two surfaces depending on the signs of the scalar products of the normals to the sliding surface and the corresponding vector fields. It is observed that the sudden onset of chaos (Fig. 2(c)) is through a grazing sliding bifurcation. Additional results demonstrating the Filippov representation to identify the discontinuity induced bifurcations and study the dynamics of nonlinear systems with discontinuous nonlinearities will be presented in the full paper.

![Figure 2](image-url)

Figure 2: (a) chaotic solution for \(\omega = 14.27\) (b) P1 solution before grazing for \(\omega = 4.56\) (c) chaotic solution for \(\omega = 4.565\).

References


Thursday Afternoon, July 9, 2015
Analytical Dynamics Based Strategy for Acceleration Control of a Car-Like Vehicle Motion

Elżbieta M. Jarzębowska

Faculty of Power and Aeronautical Engineering
Warsaw University of Technology
Nowowiejska 24 St., 00-665 Warsaw, Poland
elajarz@meil.pw.edu.pl

Abstract

The paper addresses dynamics modeling dedicated to control design for constrained mechanical systems. The constraints may originate from any source, i.e. they can be material, control based or task based, provided they can be presented in the form of algebraic or differential equations. There are quite a lot of constraints put upon engineering system models that can be presented in the equation forms. A case study example that illustrates the theoretic analytical and control developments presented in the paper is a problem of control acceleration and its change, i.e. jerk, for a car-like vehicle. The latter constraint is nonholonomic third order. The vehicle has rigid wheels in contrast to a real vehicle whose wheels have tires and the nonholonomic first order material constraints have to be transformed to the form that represents pneumatic tire characteristics. This latter work is in progress and was initiated in [3].

In a vehicle design, key factors are modelling and excluding a user discomfort, which increases with the magnitude of acceleration and jerk. Comfortable levels of these quantities have different magnitudes in the direction of motion and perpendicular to it [6]. The most common constraints put upon a vehicle performance and ride quality properties, as well as in lane changing problems, are the upon lateral acceleration and steering velocity [4], [5]. They can be written as

\[ a_{lat} = \kappa v^2 \]  

and

\[ \dot{\phi} = \frac{l}{1 + (\kappa')^2} \kappa v^2 \]  

where \( \kappa \) is the real curvature of the vehicle and \( \kappa' \) is its derivative, \( \phi \) is the steering angle, \( v \) is velocity and \( l \) is a distance between the vehicle wheels axles. It can be seen that the constraint (6) is nonholonomic. If to present a trajectory curvature \( \kappa (t) \) in the Cartesian plane (x,y), it is of the form

\[ \kappa (t) = \frac{|\dot{x} y - \dot{y} x|}{(x^2 + y^2)^{3/2}} , \]

so it is clear that the constraint (2) is third order nonholonomic.

Jerk and acceleration limits are often considered controller design objectives in car-like vehicles control design or additional vehicle performance properties. In [1], three different acceleration profiles: circular trajectory, trapezoidal acceleration trajectory, and fifth order polynomial trajectory, were used. The lateral jerk for the vehicle is specified by selecting the slope of the trapezoid, and the lateral acceleration by choosing the height of the trapezoid. Instead, we may
specify a constraint for a desired change of acceleration and jerk profiles using the constraint equations similar to (1) or (2).

Constrained dynamics that takes into account the constraints like (1) or (2) is based upon the generalized programmed motion equations (GPME) developed for systems constrained with arbitrary order nonholonomic constraints [2]. They have the form

\[
M(q)\ddot{q}+V(q,\dot{q})+D(q)=Q(t,q,\dot{q}),
\]

\[
B(t,q,\dot{q},...,q^{(p-1)})q^{(p)}+s(t,q,\dot{q},...,q^{(p-1)})=0,
\]

where \(q\) is a n-dimensional state vector, \(B\) is a \((k \times n)\) dimensional constraint matrix, \(p\) - constraint order, \(n>k\), and \(s\) is a k-vector. \(Q\) is a vector of external forces, which are not controls. Equations (3) result in a constrained dynamics that enables planning desired motion. Also, using (3) for \(p=1\), a dynamic control model already in a reduced state form, i.e. free of the constraint reaction forces, can be developed. The control dynamics is developed for a vehicle with material constraints on it only. The separation of the material and task-based constraints is a key issue that enables using analytical dynamics methods to control. The modeling framework (3) constitutes a basis for a development of an advanced control platform, which is a fusion of modern dynamics modeling, control algorithms and embedded controllers. The control platform architecture is modular and the resulted constrained system dynamics (3) enters the control loop such that it enables avoiding differentiation of control inputs.

The paper presents analytical dynamics based control strategy architecture and a subsequent controller design, however, in contrary to many results reported in literature, e.g. see [7], final control algorithms, that are passed to a control engineer, i.e. that will be applied to a real system, are the existing ones, already tested and proved having good performance. Moreover, they can be the controllers dedicated either to holonomic or nonholonomic systems, and both can be applied to nonholonomic ones. Then, a control engineer knows what sensors are to be used, what is the control convergence and tracking errors.

The paper contribution is two folded. It introduces latest analytical dynamics methods to nonlinear control and proposes methods for solving practical engineering control problems.

References


Global Dynamics of Raychaudhuri Equations

Konstantin E. Starkov*, Alexander P. Krishchenko#

* Department of Investigations, CITE DI, Instituto Politécnico Nacional, Av. IPN 1310, Mesa de Otay, 22510 Tijuana, BC, Mexico kstarkov@ipn.mx, konstarkov@hotmail.com

# Department of Mathematical Modeling, Bauman Moscow State Technical University 2-aja Baumanskaja 5, 105005 Moscow, Russia apkri@bmstu.ru

Abstract

In this work we study global dynamics of Raychaudhuri equations for a two-dimensional curved surface of constant curvature taken in the following form

\[
\begin{align*}
\dot{x} &= -\frac{1}{2} x^2 - ax - 2(y^2 + z^2 - w^2) - 2 \beta, \\
\dot{y} &= -(a + x) y - \gamma, \\
\dot{z} &= -(a + x) z - \delta, \\
\dot{w} &= -(a + x) w,
\end{align*}
\]

with real nonzero parameters. They are well-known as a model of the motion of nearby bits of matter or the evolution of deformations of a medium itself, [1-3], and exploited in the proofs of the Hawking-Penrose singularity theorems of general relativity, see [4]. Main results of our work are included in the paper [5]. They are the following.

1) We prove the nonexistence of periodic/ homoclinic/ orbits and homoclinic cycles for Raychaudhuri equations. We present formulas for equilibrium points of Raychaudhuri equations and provide local stability analysis around these equilibrium points.

2) We describe how the global behavior of trajectories is changed under varying the bifurcational parameter given by the formula \( D := \frac{1}{2} a^2 - 2 \beta - 2 \sqrt{\gamma^2 + \delta^2} \). If \( D < 0 \) then Raychaudhuri equations are free of compact invariant sets.

3) If \( D \geq 0 \) then Raychaudhuri equations possess compact invariant sets. In this case we find three-dimensional ellipsoidal domain containing all compact invariant sets which are located in threedimensional invariant plane as well. Besides, if \( D > 0 \) then there is a parallelogram located in the invariant twodimensional plane for which its vertices are equilibrium points. Its edges are heteroclinic orbits. This parallelogram bounds a compact invariant domain in this twodimensional plane. The interior of this parallelogram is filled by heteroclinic orbits connecting one pair of opposite vertices. In case \( D = 0 \) this parallelogram is shrunk into one heteroclinic orbit connecting two equilibrium points. Our analysis is fulfilled with help of the localization method of compact invariant sets, see [6,7], and properties of semipermeable and invariant planes as well.

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References


Discrete breathers in forced chains of oscillators with cubic nonlinearities

Francesco Romeo, Oleg Gendelman

*Dept. of Struct. and Geotech. Engineering
Sapienza University of Rome
Via Gramsci 53, 00197 Rome, Italy
francesco.romeo@uniroma1.it

# Faculty of Mechanical Engineering
Technion - Israel Institute of Technology
Address, Postcode City, Israel
ovgend@tx.technion.ac.il

Abstract

The forced dynamics of chains of linearly coupled mechanical oscillators characterized by on site cubic nonlinearity is investigated. The study aims to highlight the role played by the harmonic excitation on the nonlinear localised dynamics of the system. Towards this goal, a map approach is employed in order to identify the chain nonlinear propagation regions under 1:1 resonance conditions. Given the latter assumption, the governing second-order difference equation refers to a perturbation of the stationary resonant response. Therefore, at first, the map dependence on the perturbation amplitude is neglected and the dependence of the propagation regions as well as the ensuing period-1 orbits on the excitation amplitude is described. Discrete breathers (DB) obtained as map homoclinic and heteroclinic orbits are compared with analytic approximations. Simple, soliton-like solutions are identified with sequences of homoclinic or heteroclinic primary intersection points and their analytic approximation is based on the idea that the nonlinearity is taken into account only in the central part of the breather whilst the tails are treated as linear excitations.

A forced chain of linearly coupled nonlinear oscillators is studied by considering the dynamics governed by the following equation of motion for the generic $n$-th oscillator

\[ \ddot{u}_n + u_n + u_n^3 + c(2u_n - u_{n-1} - u_{n+1}) = A \cos \omega t \]  

(1)

in which $c$ represents the linear coupling stiffness and the harmonic forcing amplitude and frequency are given by $A$ and $\omega$, respectively. Time periodic solutions of equation (1) are sought for by assuming the harmonic solution $u_n = a_n \cos(\omega t)$. Equating coefficients of $\cos(\omega t)$, thereby assuming 1:1 resonance conditions, gives

\[ (1 - \omega^2)a_n + \frac{3}{4}a_n^3 + c(2a_n - a_{n-1} - a_{n+1}) = A \]  

(2)

Equation (2) describes the motion of a perturbation of the underlying stationary resonant response; by substituting $a_n = \gamma_n + \mu$ and $1 - \omega^2 = \sigma$, equation (2) leads to

\[ \sigma(\gamma_n + \mu) + \frac{3}{4}(\gamma_n + \mu)^3 + c(2\gamma_n - \gamma_{n-1} - \gamma_{n+1}) = A \]  

(3)

The cubic nonlinearity allows to set $\sigma \mu + \frac{3}{4} \mu^3 = A$; among the real roots $\mu_i, i=1,\ldots,3$, the ones corresponding to stable solutions are selected and substituted into the map

\[ \alpha \gamma_n + \beta(\gamma_n^3 + 3\gamma_n^2 \mu + 3\gamma_n \mu^2) + \gamma_{n-1} + \gamma_{n+1} = 0 \]  

(4)

where $\alpha = -\frac{\sigma}{c} - 2$, $\beta = -\frac{3}{4c}$. As known, the frequency dependent nonlinear map defined by (4) belongs to the class of area preserving maps such that $\det(D\mathbf{T}(\gamma, \mu)) = 1$, where $D\mathbf{T}$ is the Jacobian or tangent map with reciprocal eigenvalues [1].
Figure 1: Invariant manifolds scenarios for increasing forcing amplitude level; stable (blue) and unstable (red) manifolds and fixed points for $\sigma = -4.7$: a) $\mu = 0.0$; b) $\mu = 0.2$; c) $\mu = 0$.

By neglecting $\gamma_n$, the excitation effect is retained by the map through dependence on $\mu$, therefore the propagation regions boundaries can be easily identified by $|\text{tr}(\mathbf{D}_T)| = 2$, leading to the nonlinear propagation regions. Interestingly enough, the dependence on $\mu$ of the forced propagation regions coincides with that of the unforced ones with respect to the response amplitude. The presence of the forcing term alters the position of the fixed points along the symmetry line $x = y$. In essence, as the forcing amplitude level increases, the stable fixed points (blue dots in Figure 1) are no longer symmetric with respect to $x = -y$. Therefore for points lying outside the bounded regions, the fixed points $(0, 0)$ are hyperbolic and the invariant manifolds can emanate from them (see Figure 1a,b). Differently, by entering the bounded region, the fixed points $(0, 0)$ become elliptic and the unstable ones move along the main symmetry line $x = y$ (see Figure 1c). Exact analytical approaches for the analysis of DB are seldom available. In this work, following [1], discrete breathers are identified with sequences of homoclinic intersection points of the mapping corresponding to the DB centered in $n = 0$ shown in Figure 2. Then, the DB map-based analysis is compared with an analytic approach based on a single particle DB approximation and harmonic balance method [2] (Figure 2). The prediction of DB existence zone in the space of parameters provided by the two approaches is eventually discussed.

Figure 2: Comparison of discrete breathers obtained for $\sigma = -4.7, \mu = 0.2$; homoclinic orbit of $(0, 0)$ (black); single particle approximation (green).

References


Nonlinear Dynamic Structures with Developing Discontinuity

Vladimir I. Babitsky, Vikrant R. Hiwarkar

Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University,
Loughborough, Leicestershire LE11 3TU, UK,
v.i.babitsky@lboro.ac.uk

Abstract

Discontinuity is one of the most important nonlinear factors challenging both methods of nonlinear analysis and its applications. The problem becomes more complicated when the discontinuity is developing under the influence of dynamic processes in the structures.

Arising and development of the discontinuity in the structure leads to its gradual modification influenced strongly by additional nonlinear forces of impact and friction between the contact surfaces of the discontinuous elements as well as general transformation of continuum surrounding the discontinuity. This brings an essential change in dynamic response of the structure to dynamic loading which can be traced for diagnostic of the structural health to prevent the possible failure.

Several technological problems address to consideration of the structures with developing discontinuity. Among of them are: emergence of crack or delamination of material, slackening of joints, machining etc.

This paper presents the methodology for analysis and simulation of the systems with the developing discontinuity. It is based on the combination of analytical technique using nonlinear integral equations and the Matlab-Simulink computation. Introduction of the new substructure named as a virtual linear system was used to distinguish clearly the influence of geometrical and dynamical factors on the behaviour of the structure.

Gradual transformation of the structure from continuous to discontinuous one is modelling with embedding of an additional structure called as a latent defect. It includes both a nonlinear operator describing the discontinuity and a linear elastic element regulating the dynamic
response of the discontinuity. Progressive weakening of linear element stiffness controls the nonlinear influence of the discontinuity.

Concept of receptance operator along with the theory of modal analysis to generate the Matlab–Simulink model of the structures with developing discontinuities is evolved. This gives a way to strong analogy with modelling of proper feedback control systems. A procedure for convergent approximation of the modelling is developed.

This methodology is applied to analysis of nonlinear dynamics of the cracked bar. A model of a cracked bar with distributed parameters subjected to longitudinal excitation is used to analyse a nonlinear response as a way to monitor structural health. Different strategies of monitoring are compared based on the tracing the qualitative transformation of the bar’s vibration.

Some new nonlinear phenomena have been revealed and validated. It is found that there is an occurrence of the intense modulation in the time response as the crack propagates. This indicates that nonlinear transformation of the modulated signal by the crack generates a low-frequency component. Along with the side band frequencies this can be a good indicator of crack detection and its development.
Effects of a Slow Harmonic Excitation on an Atomic Force Microscope System

Faouzi Lakrad, Mourad Khadraoui, Mohamed Belhaq

Faculty of Sciences Ain Chock
University Hassan II Casablanca
BP 5366 Maarif, 20100 Casablanca, Morocco
f.lakrad@fsac.ac.ma

Abstract

The Atomic Force Microscope (AFM) [1] is one of the scanning probe microscopes. It can be used in a broad spectrum of applications such as imaging, nanolithography, electronics, chemical and biological analysis [2].

The present work is focused on the dynamics of an AFM system under the Lennard-Jones force [4] and a very slow harmonic base displacement. A single-degree-of-freedom model depicted in Figure 1 is utilized to represent a lumped-parameters model of an atomic force microscope. The cantilever-tip sample interaction is modeled by a Lennard-Jones force between a sphere and a flat surface.

![Figure 1. The AFM model. Z: the initial gap, V(t): the vertical slow harmonic displacement, X(t): the absolute displacement of the mass m, k and c are stiffness and damping coefficients.](image)

The system can be viewed as a fast-slow system with two time scales dynamics: one ruled by the natural frequency of the system and the other by the low frequency of the base displacement. We will show that solutions of the system converge towards stable slow manifolds. For more informations on the slow-fast systems see for instance [3]. In fact, two stable slow manifolds coexist, one corresponds to the contact mode and the other to the noncontact mode. These two stable slow manifolds undergo dynamic saddle-node bifurcations (through the collision with an unstable slow manifold) when the amplitude of the base displacement is varied. These bifurcations rule the operational mode of the AFM: contact, noncontact and tapping modes, respectively.
In Figure 2 is shown the computed chart of behaviors of a soft silicon microcantilever interacting with a flat silicon sample. The contact duration is found, in this case, to be 15 – 75% of the base oscillation period.

![Chart of system behaviors](image)

**Figure 2.** Chart of the system behaviors in the plane normalized amplitude $f$ of the base and the initial gap $Z$

**References**


Numerical Continuation Applied to Nonlinear Rotor Dynamics

A. Boyaci

Institute of Applied Dynamics
Technische Universität Darmstadt
Otto-Berndt-Str. 2, 64287 Darmstadt, Germany
boyaci@sds.tu-darmstadt.de

Abstract

High-speed rotors of thermal turbo-machines are usually supported in hydrodynamic bearings which may show a great variety of nonlinear phenomena. The instabilities of the synchronous oscillations due to unbalance yield several types of subsynchronous oscillations introducing various bifurcation sequences. For perfectly balanced rotor systems, the numerical continuation method is successfully applied to study the stability and bifurcation behavior of the different types of subsynchronous oscillations [1, 2]. Until now, a rotor unbalance has been mainly neglected in these investigations. Therefore, the influence of unbalance on the nonlinear dynamics is analyzed within this contribution.

To explain the fluid-induced instability mechanisms, a symmetric, linear-elastic Jeffcott/Laval rotor (mass \( m \), shaft stiffness \( c \), unbalance \( U \), phase angle \( \beta \), angular velocity \( \omega \)) is examined where single oil as well as double oil film bearings are considered (cf. Fig. 1). The equations of motion are derived with respect to a rotating reference frame \((\xi, \eta, \zeta)\)

\[
\begin{align*}
    m\ddot{\rho}_C + 2j m\omega \dot{\rho}_C + (c - m\omega^2)(\rho_C - \rho_R) &= U \omega^2 e^{j\beta}, \\
    2F(\rho_R, \dot{\rho}_R) - (c - m\omega^2)(\rho_C - \rho_R) &= 0.
\end{align*}
\]

(1)

Here, the displacements of the disk and the rotor journal are expressed in form of complex numbers \( \rho_C = \zeta_C + j\eta_C \) and \( \rho_R = \zeta_R + j\eta_R \). By assuming a short bearing, an analytical solution of the simplified Reynolds equation is used to determine the nonlinear bearing force \( F(\rho_R, \dot{\rho}_R) = F_\xi(\rho_R, \dot{\rho}_R) + jF_\eta(\rho_R, \dot{\rho}_R) \). Note that the influence of gravity is neglected. In the case of double oil film bearings (floating ring bearings), the force and the torque balance of the ring are additionally considered.

For the bifurcation analysis, the continuation tool box MATCONT [3] is used which provides methods to trace the branches of equilibrium and of periodic solutions. Then, the stability of the
obtained solutions is determined. Consequently, the types of bifurcation can be detected when (at least) one eigenvalue of the equilibrium solution crosses the imaginary axis or when (at least) one Floquet multiplier of the periodic solution leaves the unit circle.

For a small-sized Jeffcott/Laval rotor in full-floating ring bearings, an exemplary bifurcation diagram is outlined in Fig. 2 (a) where the maxima and the minima of the magnitude disk displacement |ρC| are plotted against the rotor speed \( f \). Since the equations of motion are formulated with respect to a rotating reference frame and therefore become autonomous, the periodic, synchronous oscillations in a fixed reference frame are represented by a branch of equilibrium solutions. This so-called Syn branch is unstable for low rotor speeds. However, a stable Sub i branch of periodic solutions exists in the lower speed range which is caused by an oil whirl/whip instability due to the inner oil films. By increasing the rotor speed, the Sub i branch vanishes through a Hopf bifurcation and the Syn branch becomes stable. In the higher speed range, a further Hopf bifurcation takes place at which the Sub o branch is born. In Fig. 2 (b), the normalized subsynchronous frequencies \( f_{sub}/f \) of the Sub i and the Sub o branch are given for a fixed reference frame which correspond to the whirl/whip frequencies of the inner and the outer oil films. Finally, the influence of the unbalance is investigated by tracing the Hopf bifurcations in the \((U,\omega)\)-parameter plane so that stability charts can be generated for different types of hydrodynamic bearings.

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